VALIDATING A METHOD OF MODELING A MAN-ORGANIZED SYSTEM

by

BOB B. LUKENS

MAJ, U.S. ARMY

A THESIS

Submitted in partial fulfillment of the requirements

for the degree of Master of Science in Engineering in the

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ABSTRACT

This study is intended to develop a new concept in modeling a man-organized system and to initiate implementation and testing of this model. It was desired to not only have the capability to predict the output of the organization but to also be able to characterize the internal operation of the system. These objectives may be accomplished by combining the best features of modeling from an economist's viewpoint and a control system engineering standpoint. The economists consider primarily the input and output flows and the control system theory dwells on the consideration of parameters influencing system performance. The use of the state variable equations permitted the accomplishment of these objectives.

The state variable equations were rewritten in a difference form and combined into a matrix form. Discrete data from the real-world system is used to generate a matrix which is characteristic of the system. This fitting procedure is a unique feature of the Dynamic Organizational Network Analysis (DONA) method. Multiple regression analysis is used in the DONA method to develop the system description matrix. This matrix is the primary component of the DONA model. The DONA model will produce system outputs and future system state variables given the system inputs.

Since the DONA method of producing a model is a new approach, a question arises as to its validity. The usual procedure to test the validity of a model is to use historical data from the real-world and compare the model outputs

with the results of the simulated organization. This method does not permit the desired control over the stimulus data. A laboratory concept was conceived to validate the DONA model and to discover its basic characteristics; i.e. frequency response, stability, etc. The standard for comparison was a computer simulation of a sales company. This simulation contained known operational capabilities and was therefore a good model to use for the comparison.

The complete DONA method, DONA model and laboratory concept were written into a computer program using FORTRAN language.

The results of the program pointed out some unsuspected problems in modeling a man-organized system. The computation of the system characteristic matrix leads to a better understanding of the system operation. The computer program also contains a unique feature of printing out how well the real-world system has been described in the DONA model.

Since this study was the initial effort in developing the DONA concept, there were several areas discovered that needed further study. The thesis concludes with some recommendations for further exploration.

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CHAPTER I

GENERAL MODELING

Introduction

During the past decade, significant advancements in business organization modeling have been achieved. This may be attritutable to increased interest in systems analysis and to the advent of the high-speed computer.[1*]

The concept of modeling is based on an abstraction, simplification, or idealization of the system or event. The model may be used for a variety of purposes. The most common of these is prediction or optimization of the system being modeled. Depending on the approach used in constructing the model, another extremely valuable use of the model is to describe the activity going on within the system. R. A. Brown in "Modeling the Organization"[2] discusses modeling the organization as seen from two viewpoints-economists and management. The first viewpoint is that of an accountant where the system is represented as consisting of inputs and outputs. From the economist's approach, very little is said about what occurs internal to the system. The second viewpoint is that of the control system engineer. The system is viewed from within. Control of the organization is attempted through the manipulation

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* Numbers in superscript throughout the thesis indicate the reference as listed in the List of References.

of selected internal characteristic quantities to have the greatest effect on the system performance. Then the control systems approach to modeling accomplishes the desired goals of modeling, predicting future events and optimizing system performance, and further permits an understanding of the endogenous (internal) variables that control the operation of the organization.

The Problem

With the understanding of why the modeling of an organization is performed, it is pertinent to direct attention to one type of an organization--the man-organized system. The man-organized system is characterized by elements such as people, material, money, and information. The set of elements together with the relationships, define the man-organized system.

The man-organized system to be considered in this thesis is a large governmental agency. In general, the method of modeling developed herein is not restricted to this type of organization. It will be seen that the method may be applicable to other organizations.

In the modeling of this large governmental agency, it was required that the modeling effort be performed in a reasonably short time. The model must characterize the system through use of elemental quantities related to basic system operation. Every variable in the system need not be accounted for in the model. This results in a gross representation of the system and this was the desired approach.

Before pursuing a new approach to the problem, it is informative to

note how the problem is currently being handled. The objectives are to know something about, (1) how the system will behave in the future given that estimates are possible of the workloads applied to the organization, (2) what parameters in the system have the greatest control over the system performance and (3) what is the magnitude of system variation with a known change in the significant parameters. The usual procedure is to collect information from selected monitoring points in the system and assemble this information into a report, usually a recurring report. Using the data from these reports, an analysis is made and an experience factor is applied to estimate the internal system characteristics and the system outputs.

Another method used in attacking the problem and searching for an answer to the objectives is to model the organization. Modeling permits the representation of the system in quantifiable terms. However, the complexity of a large organization makes it very difficult to represent the system in mathematical form. Further, the solution of the mathematical expressions may be even more elusive. The concept of computer simulation has been used in solving this problem. One of the most extensive works on computer simulation of large organizations is by Jay W. Forrester.[3] Forrester introduces a different concept in modeling large organizations. He uses information feedback ideas to describe "industrial dynamics," his concept of dynamic modeling. Forrester also develops his own computer language, known as DYNAMO, to use in his simulations. The Industrial Dynamics method of modeling is suited to an effort which requires detailed representation of a system. This method may require years for accomplish-

ment of the objectives.

Since it was desirable to improve upon the current methods of collecting and analyzing reports or using the lengthly Industrial Dynamics simulation procedure, a search began for a new method of modeling that would be timely and lend to the understanding of the performance of the man-organized system.

This thesis will give insight to a new method of modeling and develop the concept into the early stages of computer program implementation and testing. Chapter II gives an example of a computer simulation of a large system. This simulation shows that mathematical expressions are required to use the simulation method. This simulation was included to illustrate the complexity of this approach even for a small organization. Chapter III introduces the new concept for modeling. The Dynamic Organization Network Analysis (DONA) method is developed in this chapter. Chapter IV discusses a unique procedure in validating a model. The laboratory concept for the testing of a system model is introduced. Chapter V gives the results of this study and provides areas for further study in the use of the DONA method.

CHAPTER II

MAN-ORGANIZED SYSTEM AS SIMULATED WITH A COMPUTER MODEL

Design and Development Considerations in Simulation

This chapter describes a computer model which is used to simulate a man-organized system. This model serves two purposes. It serves to illustrate the detail and logic needed to construct a computer model and it will be used in the validation procedure described in further chapters.

"Modeling" and "simulation" are often times used interchangeably. There is a difference in these terms. The model as used herein means the set of mathematical equations which quantitatively describes the system being modeled. A computer model is the mathematical model written in a form to be used by the computer. Then simulation is the use of the model. The methodology of simulation begins with constructing the mathematical model. In describing a large governmental agency of a thousand or more people in a quantitative form the task may become quite formidable. In fact the expression of certain actions in mathematical equations may be elusive or impractical. The detail required for such a characterization will be illustrated in the model described below. The model will assume certain stochastic processes. Although the model is hypothetical, these statistical quantities could be verified if the model were really describing a real system; i.e., the probability distribution and its mean and variance of a particular variable could be derived from the real system.

Another assumption in the hypothetical model is that the policies of the organization are correctly defined mathematically. Of course, one use of the model is to test the optimality of the policies. The assumptions and policies are used to bound the problem. This model is therefore an effort to reduce a complex system to manageable proportions.

Description of SIMCO

The computer model is a modified model that was originally developed by McMillan and Gonzalez.[4] This model is known as SIMCO and is written in FORTRAN language. It is a model for a total system with specified parameters. The total system is a business firm which sells one product. It does not manufacture the product but does order it from the supplier. In this sense SIMCO is a sales distributor and is characterized in general as a warehousing situation. Customers place demands in a random manner and their orders may be filled immediately or not filled depending on availability of stock. The supplier requires lead time for delivery which is also a random process. The business activity is simulated on a weekly basis. The model performs calculations of the activities at the end of each week and stock is received at the end of each week.

The computer program describes the random processes and company policies of SIMCO first. These processes and policies are described as follows.

Lead time for delivery from the supplier varies from four weeks to eight

weeks with the following probability distribution:

Lead Time (weeks)	Probability
4	.10
5	.15
6	.50
7	.15
8	.10

Customer demand is uncertain from week to week but is normally distributed with a mean of 50 units per week and a standard deviation of 15 units.

SIMCO management allows a price markdown if weekly sales have not been deemed satisfactory. The normal retail price is $100 per unit; however, if demand during the week was less than 70 per cent of forecasted demand, the retail price is marked down to $85. The price markdown is also dependent on another condition. The inventory on hand at the end of the week plus new stock scheduled for delivery over the weekend must be more than 150 per cent of the forecasted demand for the coming week. This "overstock" condition will cause price markdown. Forecasted demand is found by averaging the demand during the past three weeks. Experience shows that the price markdown has caused a sales stimulation of 5 per cent.

At the end of the week SIMCO determines the activity of the past weeks and then places its order for new stock. The amount to order is determined by several factors, one of which is the desired inventory level. The desired inventory is three times the demand forecast for the coming week. The

amount to order is the sum of the quantity of units sold during the past week, one half of the difference between the desired and actual inventory, and one half of the difference between the desired and actual supply backorder. The desired supply backorder is the expected demand during the lead time, based on the demand forecast for the coming week.

SIMCO's experience has shown that about 20 per cent of the backorders are lost if they are not filled the following week. Thus a backorder situation is permitted in SIMCO.

The wholesale cost for units remains constant at $50 per unit. The payment is made immediately after receipt of the new stock.

McMillan and Gonzalez continue their description of SIMCO by next looking at the money transactions. There are two types of costs, fixed and variable. Fixed costs are $2,000 per week. Variable costs are 20 per cent of the investments in inventory at the beginning of the week, at a yearly rate, and 8 per cent of the investment in accounts receivable at the beginning of the week, at a yearly rate. The fixed and variable costs are cash costs and represent an outflow of cash at the end of each week. Cash inputs to SIMCO occur with the sale of the units. Again it has been observed that one half of the sales are in cash and the other half are on credit. The charge sales are payable four weeks later and do not incur a service charge.

SIMCO Sequence of Operations

The sequence of operations for simulating 100 weeks operations is shown in the Flow Diagram, Figure 2.1. There is a main program with four subroutines. Initialization occurs in the main program and the subroutine supply is called to determine how much new stock was received over the weekend. The demand subroutine is then called. This segment simulates the demand that SIMCO will experience in the forthcoming week. As McMillan and Gonzalez have constructed SIMCO, all cash transactions occur next and these occur in the main program. The computer model concludes with a subroutine to forecast demand and a subroutine to determine the quantity of units to be ordered. The entire program is put in a loop which repeats 100 times, each loop representing one week of operation.

SIMCO Initial Conditions and Programs Elements

The programs elements as summarized above will now be described in more detail. Before doing this, the initial conditions of SIMCO are considered. Any time that a simulation is run on the computer, there is the problem of starting conditions and equilibrium.[5] If the starting conditions vary greatly from the steady state conditions, the time to reach stabilization may be quite long. It is important to reach steady state conditions because the information content of the simulation is contained in the equilibrium state. The transient state may be greatly shortened by the choice of reasonable starting conditions. The following initial conditions were used in the SIMCO model.

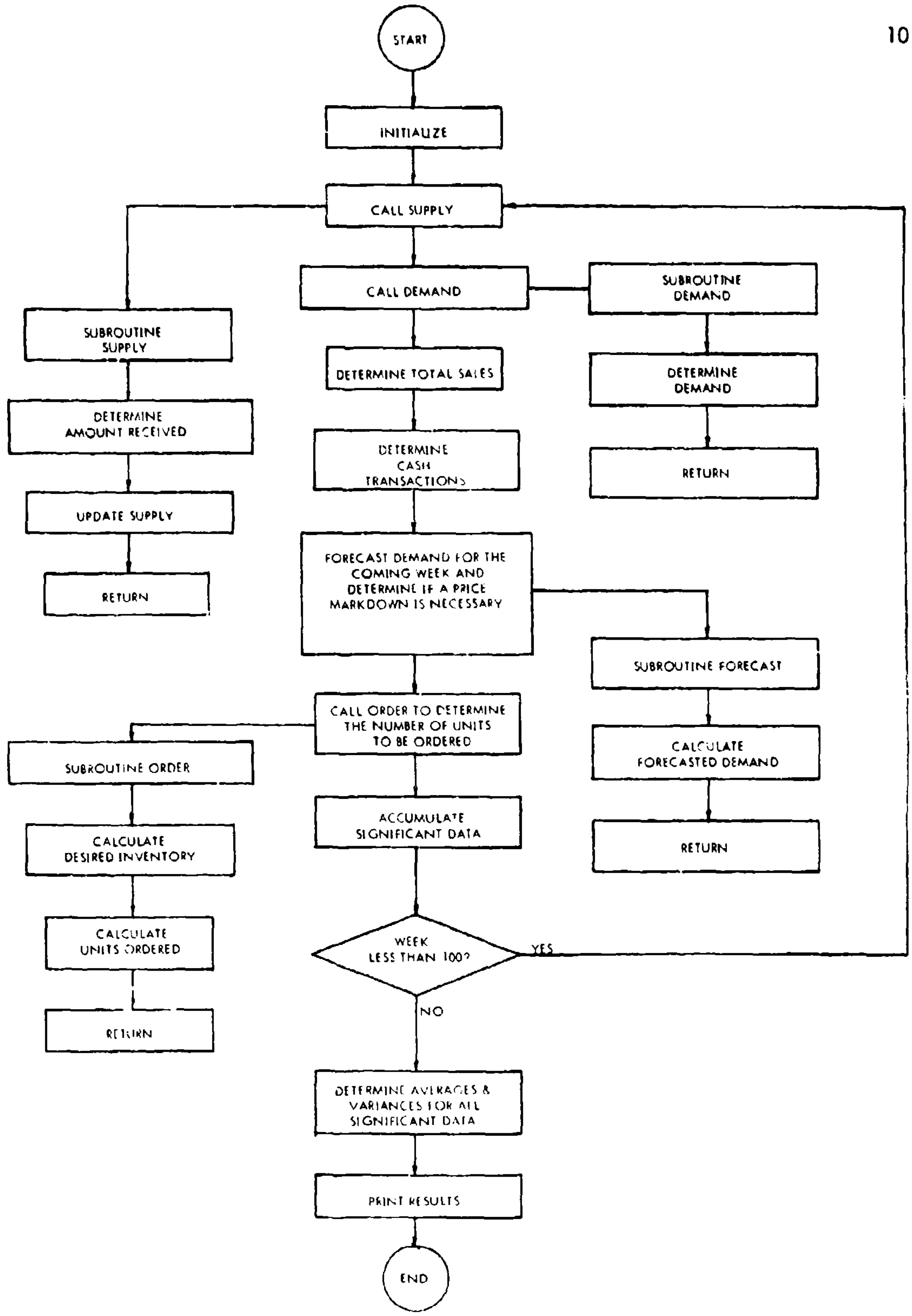


Figure 2.1 Flow Diagram of SIMCO

Demand during the past 10 weeks has been 50 units each week;

SIMCO is destined to receive, on each of the coming six weekends, 50 units of new stock;

The actual inventory on hand is 150 units;
Demand forecast for the current week is 50 units;
There are no customer backorders outstanding;

There is no price markdown planned for the current week;

$2,500 is scheduled to be received from accounts receivable collections over each of the coming four weeks (thus total accounts receivable are $10,000);

Cash on hand stands at $4,000.

It was stated that the Supply subroutine was called to determine the amount of stock received over the weekend. Since orders have already been placed, SIMCO new stock may be "destined to be delivered" in 1, 2, 3, 4, 5, 6, 7, or 8 weeks. These simulated deliveries in the pipeline will be designated by the variable, PIPLIN(K). A random number generator will simulate the lead time according to the probability distribution listed above. Subroutine Random produces two random numbers. The random numbers, RND2, is uniformly distributed from 0 to 1 and is used in the Supply subroutine. The random number DEVIOT, is normally standard distributed (mean = 0, standard deviation = 1.) and is used in the subroutine Demand. The quantity of units ordered, UO, of the Order subroutine is applied to the lead time. In effect,

the Supply subroutine creates a delay, an uncertain delay, in receiving the units ordered. The output of the Supply subroutine is the stock received, SR. The main program takes the stock received and fills the customer backorders, if any. An inventory updating occurs by adding to the actual inventory at the end of the previous week, AIEW, the stock received over the weekend to yield the actual inventory beginning the new week, AIBW. The customer backorder queue, CBQ, is subtracted to yie d the adjusted actual inventory beginning the new week, AAIBW. At this point, units of lost sales, ULS, may occur if the customer backorders exceed the inventory, a stockout condition. This condition is noted in the program.

The main program calls the subroutine Demand to determine the demand which SIMCO will experience during the coming week. A random number generator produces the random numbers according to a normal distribution and the demand is simulated. Using the price markdown, if any, which was determined in the main program through consideration of the management policy for the price markdown, the subroutine Demand applies the sales stimulation factor, SSF, to increase the simulated demand by 5 per cent.

The main program then begins a series of update transactions. Using the current week's demand, ADDW, the inventory on hand after customer backorders were filled at the beginning of the week, AAIBW, and backorder sales at the beginning of the week, BS, the following are calculated:

Total Sales during the week, TUS

Actual Inventory at the end of the week, AIEW

Gross Profit and Gross Income from the week's sales, WGP and GIS, respectively.

The main program continues with the money transactions. The Cash on Hand, COH, will be increased by accounts receivable collected during the week and income for cash sales during the week. COH is decreased by payment for new stock, fixed charges of $2000 per week, and variable costs. The variable charges consist of the inventory holding charge and the investment charge on the total accounts receivable. The accounts receivable for a particular week is one-half the gross income from sales four weeks past. The total accounts receivable is the sum of accounts receivable from sales 1, 2, 3, and 4 weeks past. The weekly net profit, WNP, is then determined from the weekly gross profit minus the operating costs (fixed costs and variable costs).

The demand forecasting subroutine merely takes the demand during the most recent past three weeks and averages them to produce the forecasted demand for the coming week. This process of determining the forecasted demand can be recognized as a moving-average, smoothing operation.

The main program uses the actual demand during the week and the forecasted demand to implement the management's policy of declaring or not declaring a price markdown.

The Reorder subroutine determines the number of units to be ordered from the supplier at the end of the week. Knowing that the desired inventory is three times the forecasted demand, the difference of actual inventory and desired inventory is determined as the first element of the decision to reorder.

The second element is based on the difference of the desired and actual supply backorder pipeline. The quantity of units to order is then the total units sold during the week plus one-half the sum of the two differences noted above.

The SIMCO total systems simulation is concluded with a report generating sequence. McMillan and Gonzalez recognized the need to produce something that could be used to analyze the changes in total operation when testing the value of a single management policy. To this end the main program contains a series of statements to determine averages and variances for all significant data. These are printed out for analysis.

The complete SIMCO computer listing is shown in Appendix A. The output from this simulation is also shown in Appendix A.

Modification to SIMCO

It is readily apparent that the SIMCO model described above is extremely rich in its formulation and operation. The model contains a wealth of features related to mathematical operations. These are queues, lags, smoothing averages, and forecasting built into the model. However, the use that this model is to be put to later in this project requires some additional features. SIMCO is a single product model. Since it is a single product model, it obviously has no interactions with other products and this is desired. SIMCO does not contain personnel considerations. It is desired to simulate personnel through consideration of skills and numbers of personnel. With this in mind, extensive modifications were made to SIMCO as shown in the Flow Diagram in Figure 2.2.

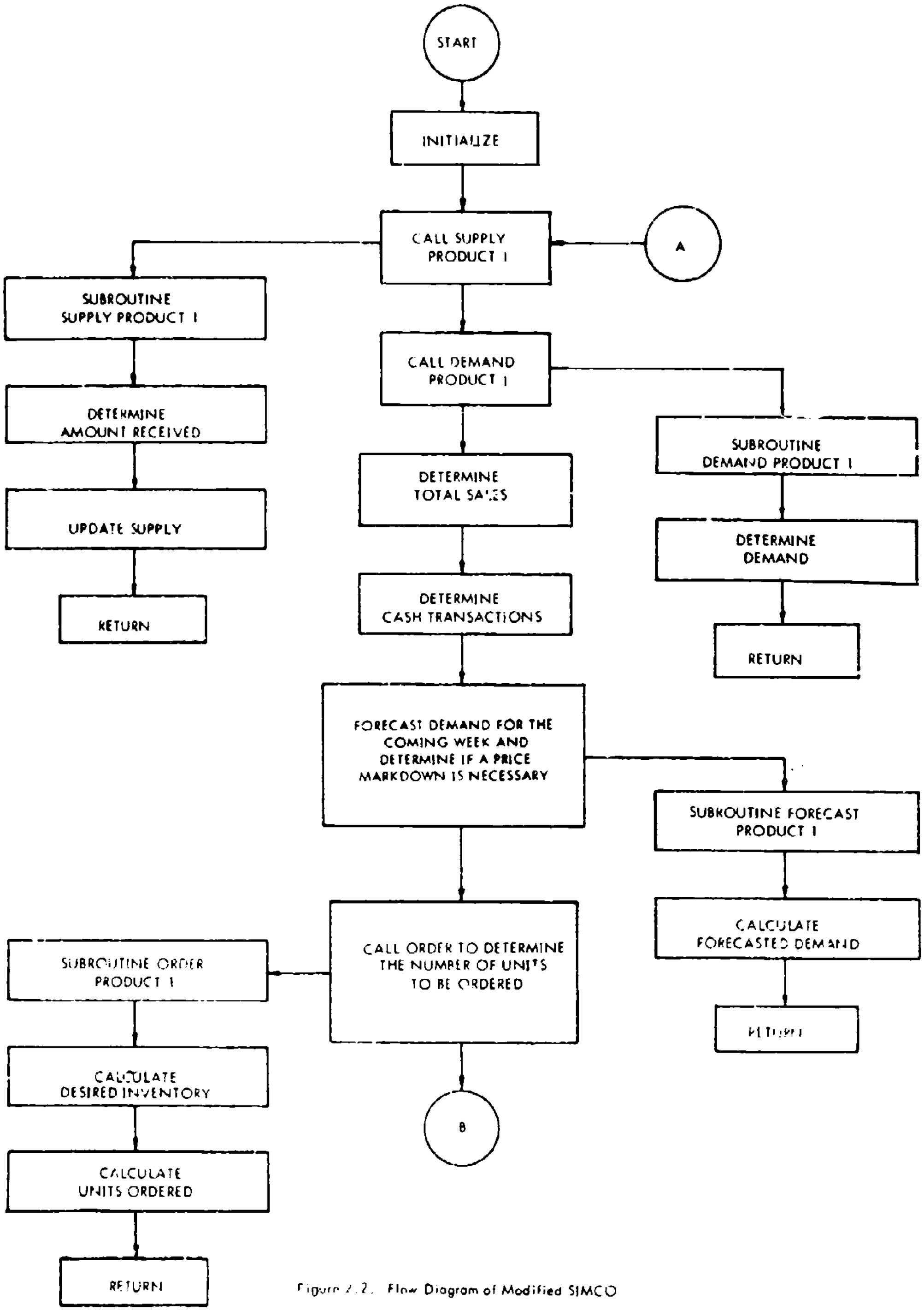


Figure 2.2. Flow Diagram of Modified SIMCO

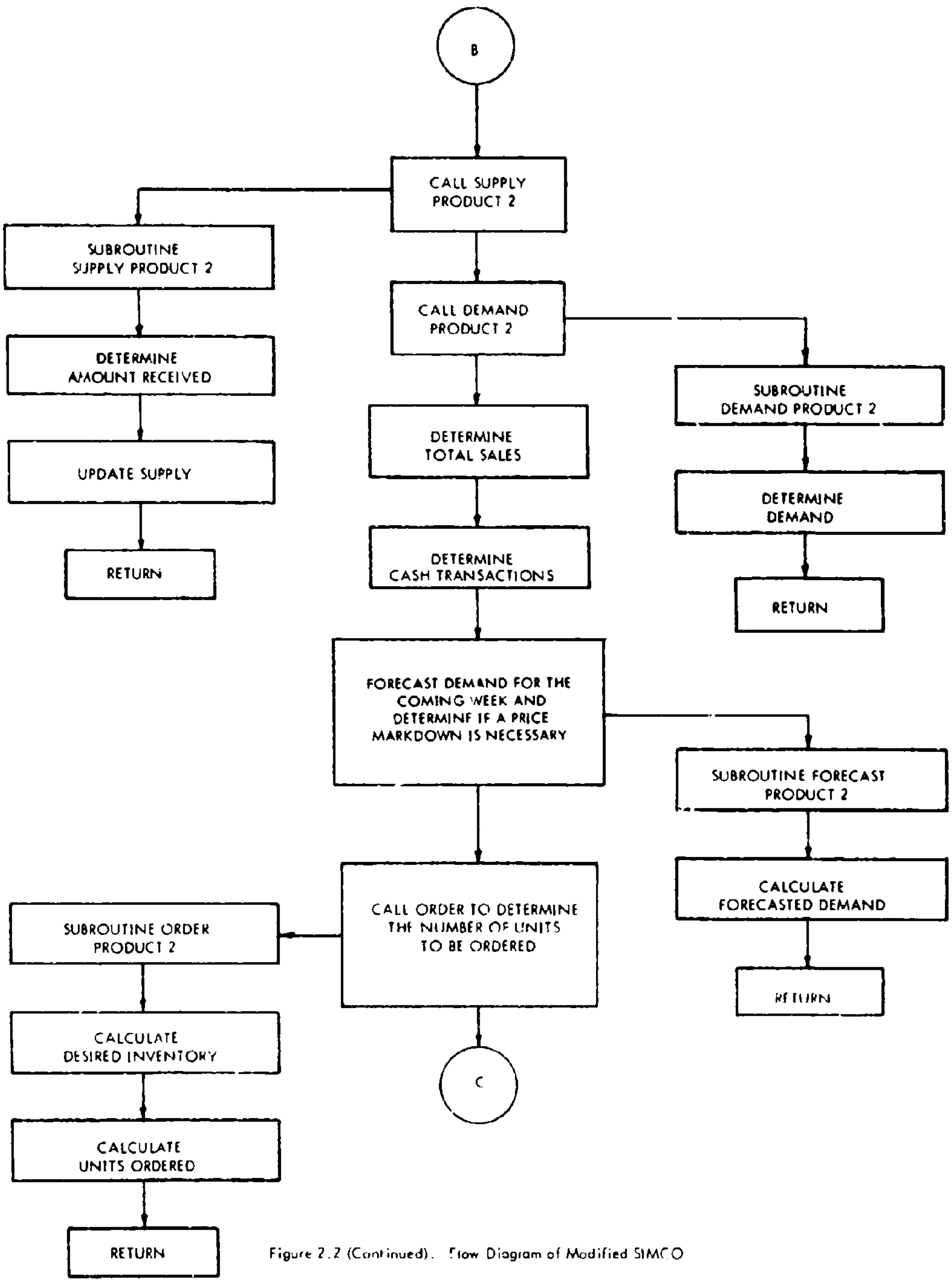


Figure 2.2 (Continued). Flow Diagram of Modified SIMCO

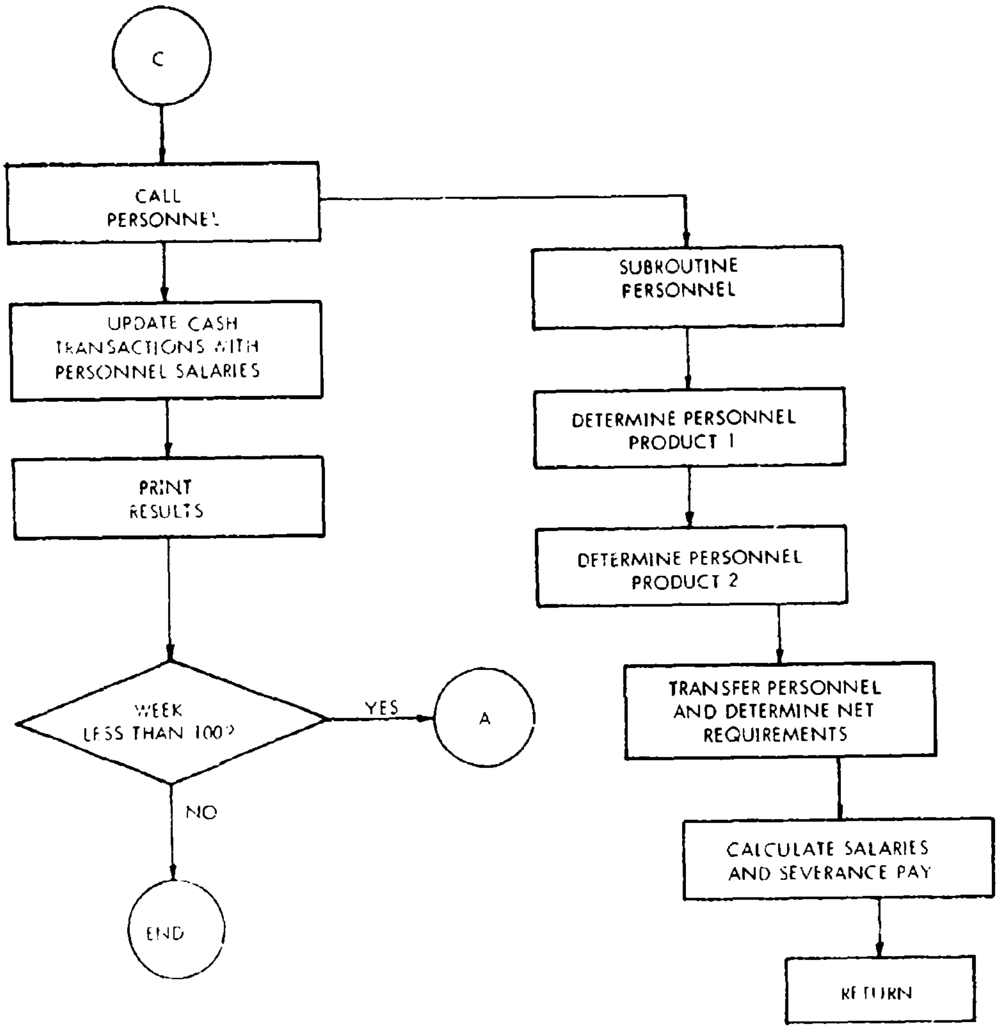


Figure 2.2 (Continued). Flow Diagram of Modified SIMCO

A second product line was added to the SIMCO model. All four subroutines of Supply, Demand, Forecasting, and Reorder had to be developed for the second product. Now customers can place demand for product one and product two, or both. The demand characteristics may be the same or different. The modification will permit independent operations of the two products.

A completely new subroutine was developed for personnel considerations. A hypothetical staffing and manning organization was used to base the skills and numbers of personnel. Part of the original SIMCO fixed costs were allocated to salaries for the SIMCO manager and administrative personnel. These costs and personnel remained fixed regardless of company operations. However, additional personnel were added as salesmen. The salesmen were assumed to have a certain productivity; i.e. the salesmen had the capability of selling up to 20 units per week of their particular product. The number of salesmen for each product was based of the forecasted demand for the coming week. This resulted in a hire-fire operation. But to make the model a little more realistic, the fire procedure caused SIMCO to pay two weeks salary for severance pay.

The personnel subroutine provided an excellent opportunity to bring in some interaction between the product lines. The personnel subroutine was further developed to allow transfer of salesmen from one product line to the other. This permits a salesman to be transferred from a product whose demand is forecasted low to a product whose demand is forecasted high. Personnel

actions now include hire-fire-transfer actions. This policy has desirable side effects. Rather than having such high variations in hire-fire, the added transfer policy has dampened these variations and has resulted in a more realistic company simulation.

Modified SIMCO Results

The modified SIMCO may be used in the same manner as the original SIMCO. The modified SIMCO may be used to test various management policies. Since it was not desired to use the modified SIMCO for this purpose, this was not pursued. The flow diagram and the complete computer listing for the modified SIMCO is shown in Appendix B. It is sufficient to note that the modified SIMCO did produce results to verify that the modifications were effective in that a second product line could be simulated independent of the first product line except for the desired interaction of personnel considerations. The personnel modification further expanded the usefulness of the simulation.

CHAPTER III

DYNAMIC ORGANIZATIONAL NETWORK ANALYSIS

Introduction

This chapter will describe the Dynamic Organizational Network Analysis (DONA) method and will describe the use of this method as applied to modeling the man-organized system.

The DONA method is a new approach to the modeling of socio-economic systems. The model is derived from control theory considerations much the same as used by the electrical engineers for analysis of systems. The strength of this approach lies in the fact that the system is considered from both the economic and management view point. The method for producing the DONA model adopts the regression technique from the economic models and preserves the multi-dimensionality of the management models.[2]

The control theory approach will be exploited in the manner described by H. E. Koenig[6,7,8,9] when he used the state variable equations to model a socio-economic system, specifically a university.[10] Koenig, et.al., in his text "Analysis of Discrete Physical Systems"[8] stated, "Machine simulations as well as recent advances in the theory of stability, control, optimization, design, and synthesis are all based on what are referred to as state-space models." The state-space model concept is the basis of the DONA model. The approach applies to linear systems but this limitation does not invalidate its application to non-linear systems.

This thesis will extend the extensive work by Koenig by applying the

regression technique to discover the "structure" of the system to be modeled. Koenig derived the structure of the university[10] through the lengthly process of writing the mathematical equations for each activity within the university. He admits that all variables were not quantifiable. It will be shown that the DONA model does not require the representation of the system as a set of mathematical equations. The structure of the system can be determined from observations by regression methods. The advantages of the DONA model will lie in the ease of estimation of the required constants, representing the structure, because of the existence of an objective method for determining those constants. The resulting representation will be meaningful because it can be related to control theory, which will give guidance for system improvements.[2]

The State-Space Approach

In using the state-space approach[8,11] to systems analysis, it is necessary to be able to represent the system with equations. Such a representation is the mathematical model and forms the starting point for the problem of system analysis. A block diagram representation for a system is shown in Figure 3.1 .

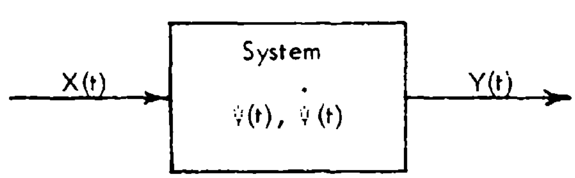


Figure 3.1 Block Diagram of a System

In Figure 3.1 the $X(t)$ is a vector representing the inputs or stimulii and the $Y(t)$ is a vector representing the outputs or responses. The $\psi(t)$ and $\dot{\psi}(t)$ are vectors representing the state variables and the first derivative of the state variables, respectively. Then the specification of the values of the state variables at some time instant is the <u>state</u> of the system at that instant.

Assuming a general n-th order linear system, the mathematical model is formulated from a set of simultaneous first-order differential equations and a set of simultaneous algebraic equations. Differential equations to represent the general n-th order, linear, time-varying system for a single input and output, can be written as

$$\frac{d\psi_1(t)}{dt} = p_{11}(t)\,\psi_1(t) + p_{12}\,\psi_2(t) + \dots + p_{1n}(t)\,\psi_n(t) + q_1(t)\,x(t)$$

$$\frac{d\psi_2(t)}{dt} = p_{21}(t)\,\psi_1(t) + p_{22}(t)\,\psi_2(t) + \dots + p_{2n}(t)\,\psi_n(t) + q_2(t)\,x(t) \qquad (3\text{-}1)$$

$$\vdots$$

$$\frac{d\psi_n(t)}{dt} = p_{n1}(t)\,\psi_1(t) + p_{n2}\,\psi_2(t) + \dots + p_{nn}(t)\,\psi_n(t) + q_n(t)\,x(t).$$

The output equation is in algebraic form

$$y(t) = m_1(t)\,\psi_1(t) + m_2(t)\,\psi_2(t) + \dots\ m_n(t)\,\psi_n(t) + n(t)\,x(t) \qquad (3\text{-}2)$$

In equations (3-1) and (3-2) the $p_{ij}(t)$, $q_i(t)$, $m_i(t)$, and $n(t)$ are time-varying coefficients of their respective variables.

The above equations considered only a single input and single output.

However, it is possible to represent a system with several inputs and outputs by a set of equations by adding to each equation the appropriate contribution from each input. Similarly, the output equation is expressed in terms of the state variables by a set of equations instead of a single equation.

For a large complex system, there is a need for a compact notation when there are many inputs, outputs, and state variables. Through the use of vectors and matrices, the equations may be reduced to a compact form.

If there are n state variables, these can be combined in a column vector, or an $(n \times 1)$ matrix, and designated as $\underline{\psi}(t)$. Thus,

$$\underline{\Psi}(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \vdots \\ \psi_n(t) \end{bmatrix} \quad (n \times 1) \tag{3-3}$$

is known as the state vector. If there are m input signals, the input vector will be designated as

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} \quad (m \times 1) \tag{3-4}$$

Likewise, if there are p output signals, the output vector is

$$Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix} \quad (p \times 1) \tag{3-5}$$

This thesis will consider only the time-invariant system which results in constant coefficients in the equations. The coefficients of the state variables can be represented by an $(n \times n)$ matrix.

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & & & \\ p_{n1} & \cdots & \cdots & p_{nn} \end{bmatrix} \quad (n \times n) \tag{3-6}$$

The matrix to represent the input coefficients is

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & & & \\ q_{n1} & \cdots & \cdots & q_{nm} \end{bmatrix} \quad (n \times m) \tag{3-7}$$

For the output equations, the matrix of constant coefficients for the state variables is

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & & & \\ m_{p1} & \cdots & \cdots & m_{pn} \end{bmatrix} \quad (p \times n) \qquad (3\text{-}8)$$

The final matrix represents the input coefficients which are a part of the output equations.

$$N = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1m} \\ n_{21} & n_{22} & \cdots & n_{2m} \\ \vdots & & & \\ n_{p1} & \cdots & \cdots & n_{pm} \end{bmatrix} \quad (p \times m) \qquad (3\text{-}9)$$

By substituting the above defined vectors and matrices into equations (3-1) and (3-2) the following equations result and are known as the state variable equations.

$$\dot{\underline{v}}(t) = P\,\underline{v}(t) + Q\,X(t) \qquad (3\text{-}10)$$

$$Y(t) = M\,\underline{v}(t) + N\,X(t) \qquad (3\text{-}11)$$

or

$$\begin{bmatrix} \dot{\underline{v}}(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} P & Q \\ M & N \end{bmatrix} \cdot \begin{bmatrix} \underline{v}(t) \\ X(t) \end{bmatrix} \qquad (3\text{-}12)$$

The use of the state variable equations will require the equations to utilize discrete information. This will require a difference form of the state

variable equations. The continuous form of the equations (3-12) are converted to the difference form as follows

$$\frac{\psi(t+h) - \psi(t)}{h} = P\,\psi(t) + QX(t) \qquad (3\text{-}13)$$

since

$$\frac{d[\psi(t)]}{dt} = \lim_{h \to 0} \frac{\psi(t+h) - \psi(t)}{h}$$

$$\psi(t+h) = hP\,\psi(t) + \psi(t) + hQ\,X(t) \qquad (3\text{-}14)$$

Then

$$\begin{bmatrix} \underline{\psi}(t+h) \\ \underline{Y}(t) \end{bmatrix} = \begin{bmatrix} (hP+I) & hQ \\ M & N \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}(t) \\ \underline{X}(t) \end{bmatrix} \qquad (3\text{-}15)$$

letting h = 1

$$\begin{bmatrix} \underline{\psi}(t+1) \\ \underline{Y}(t) \end{bmatrix} = \begin{bmatrix} (P+I) & Q \\ M & N \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}(t) \\ \underline{X}(t) \end{bmatrix} \qquad (3\text{-}16)$$

Define the S matrix,

$$\begin{bmatrix} S \end{bmatrix} = \begin{bmatrix} (P+I) & Q \\ M & N \end{bmatrix} \qquad (3\text{-}17)$$

and substituting

$$\begin{bmatrix} \underline{\psi}(t+1) \\ \underline{Y}(t) \end{bmatrix} = \begin{bmatrix} S \end{bmatrix} \cdot \begin{bmatrix} \underline{\psi}(t) \\ \underline{X}(t) \end{bmatrix} \qquad (3\text{-}18)$$

This equation is in a form that suggests the use of multiple regression analysis (see equation (10) in Appendix C). In fact the S matrix may be determined if regression methods are applied to the system to be modeled. If equation

(3-18) were used to model the system, it is suspected that this would predict system performance in both short term and long term time durations. These characteristics may be desirable in some modeling problems but this study is more concerned with short period predictions, especially because of the assumed constant-coefficient form of the equations. Another form of the state equations appears to be applicable to the short term prediction. Further study is needed to understand the characteristics of using the model in equation (3-18) and the short term model to be described below. It will be based on differences of a variable from one time period to another. This is equivalent to high-pass filtering of the data.[12]

The term "forward difference" refers to the variable's value in one time period in advance minus the variable's current value. In mathematical form the forward difference, $\Delta(x)$, is

$$\Delta(x) = x(t + 1) - x(t). \tag{3-19}$$

Similarly, the backward difference is the variable's current value minus its value one time period in the past. The backward difference, $\nabla(x)$, is

$$\nabla(x) = x(t) - x(t - 1). \tag{3-20}$$

Manipulation and substitution of equation (3-19) and (3-20) into equation (3-18) yields

$$\begin{bmatrix} \Delta(\Psi) \\ \nabla(Y) \end{bmatrix} = \begin{bmatrix} S \end{bmatrix} \cdot \begin{bmatrix} \nabla(\Psi) \\ \nabla(X) \end{bmatrix} \tag{3-21}$$

The methodology for developing the DONA model is succinctly summarized in equations (3-19), (3-20), and (3-21). The forward and backward differences are determined for the appropriate variables selected from the system to be modeled. Linear algebra is used to solve the matrix equation (3-21) for the S matrix and this is the matrix which characterizes the system.

The DONA model is represented by equation (3-18). Having determined the S matrix as described above, the DONA model is determined.

CHAPTER IV

THE LABORATORY EXPERIMENTAL SYSTEM

Validating The Model

In the previous chapters, two completely unrelated types of models for man-organized systems were shown. A new modeling procedure based on the state-space approach was discussed and a computer simulation of an organization was explained. This chapter will show a way to validate a model which is quite different from the "classical" method of validating a model. It will be seen that this new procedure provides many important advantages over the usual model validation procedure.

Validation or verification, as it sometimes called,[1] is a procedure used to determine the "truth" or "worth" of a model. Then the concept for validation is based on establishing a set of criteria for differentiating between what is true and what is not true. This is not unlike the problem of testing a statistical hypothesis. However, the procedure or the method for validation is the question. The most common method, the "classical" method, of validating the model is to use data from the real world, which the model is simulating, and observe the performance of the model. As pointed out in Chapter I, a model may serve two purposes. It may be used to determine optimal policies and operation of a system or it may be used to predict the performance of the system. Naylor, et. al.,[1] in discussing the problem of verification said:

> It is our position that the ultimate test of a computer simulation model is the degree of accuracy with which the model predicts the behavior of the actual system (which is being simulated) in the future.

This statement is true but it does not fulfill the requirement for validating the DONA model. An immediate requirement exists for testing the DONA model and the method for producing the DONA model to discover some of the basic characteristics. This requires control over the variables being applied to the model. The actual system cannot be manipulated to produce the range and magnitude of variables needed to discover the operating characteristics of the DONA model. For this reason, a laboratory concept was developed to validate the method. In this "laboratory," the parameters can be controlled to determine the range and responsiveness of the model. The laboratory concept was used and will be described in the following sections.

The Laboratory System

The concept for validation of the DONA Model and hence the quality of the valid method used to produce the DONA model was to use SIMCO to simulate the "real world" organization and implement the DONA methodology to model SIMCO. SIMCO was used because its characteristics were known. Using the same inputs to SIMCO and DONA the validity of DONA can be estimated by comparing the outputs of the two models. The complete "laboratory" system is shown in Figure 4.1. The implementation of this laboratory was through the use of a computer program in FORTRAN V language to describe the elements

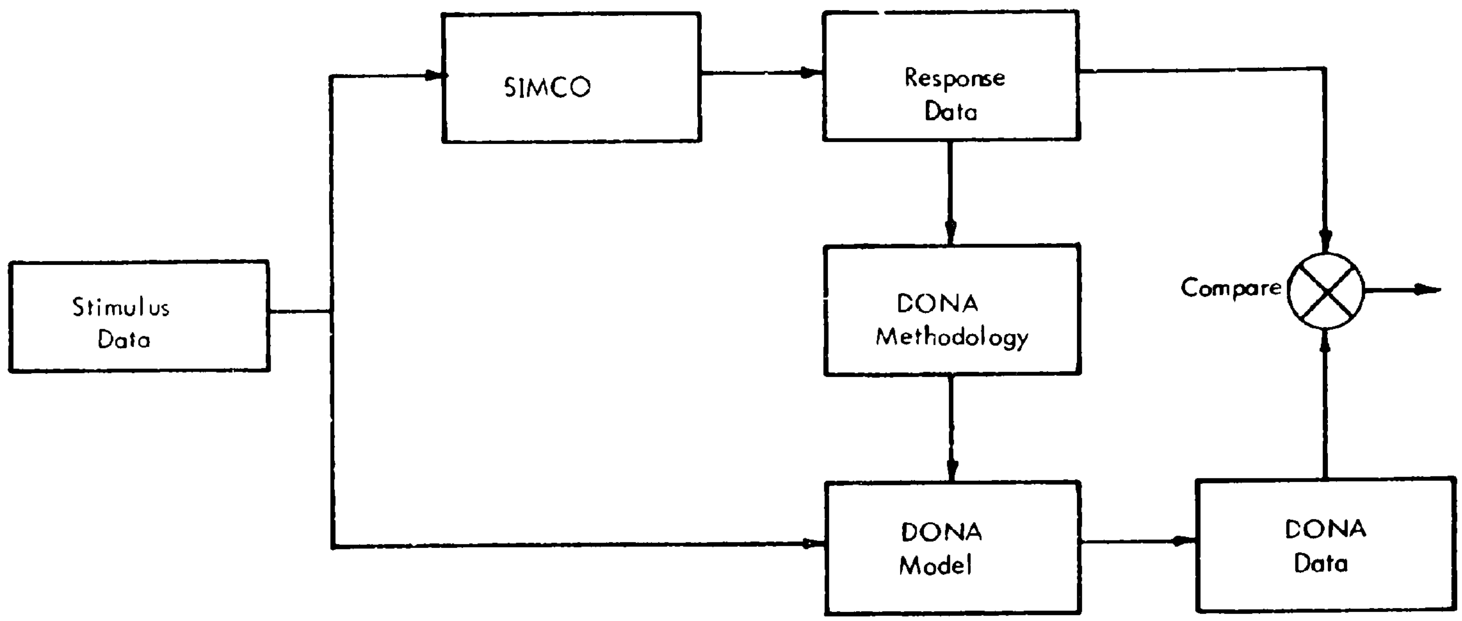


Figure 4.1 The Laboratory System

and flows in the laboratory. The computer-based laboratory system in Figure 4.2 will now be described in detail.

SIMCO was described in detail in Chapter II but some additional considerations had to be employed to use the simulation in the laboratory system. SIMCO produces about 50 variables. Each of these variables had to be analyzed to group them in one of three types of variables--input, output, or state variables. Even though the SIMCO simulation , per se, is unconcerned about which variable is which, the identification of these variables for the state-space approach is very important.

It is not sufficient to draw an imaginary line around SIMCO and identify everything crossing the line as either an output or input. This is the accountant's viewpoint of identifying the flows as inputs or outputs. The control theory viewpoint must also be exploited. The system must still be defined. The same imaginary line may be used, but now the terminals must be considered as either a propensity variable or flow rate variable depending on its characteristics. This analysis performed on SIMCO yielded some rather unexpected results. Even though there were many variables in SIMCO, the terminal analysis showed that there were not enough variables produced by the simulation. Some equations were then introduced in the simulation to complete the terminal requirements. An example of this problem is the quantity of stock received each week, a flow rate variable, from the supplier. The stock received is an input to SIMCO; however, there is no singularly identifiable propensity at the output which is complementary to this input. A new output variable, wholesale cost, was developed to account for the com-

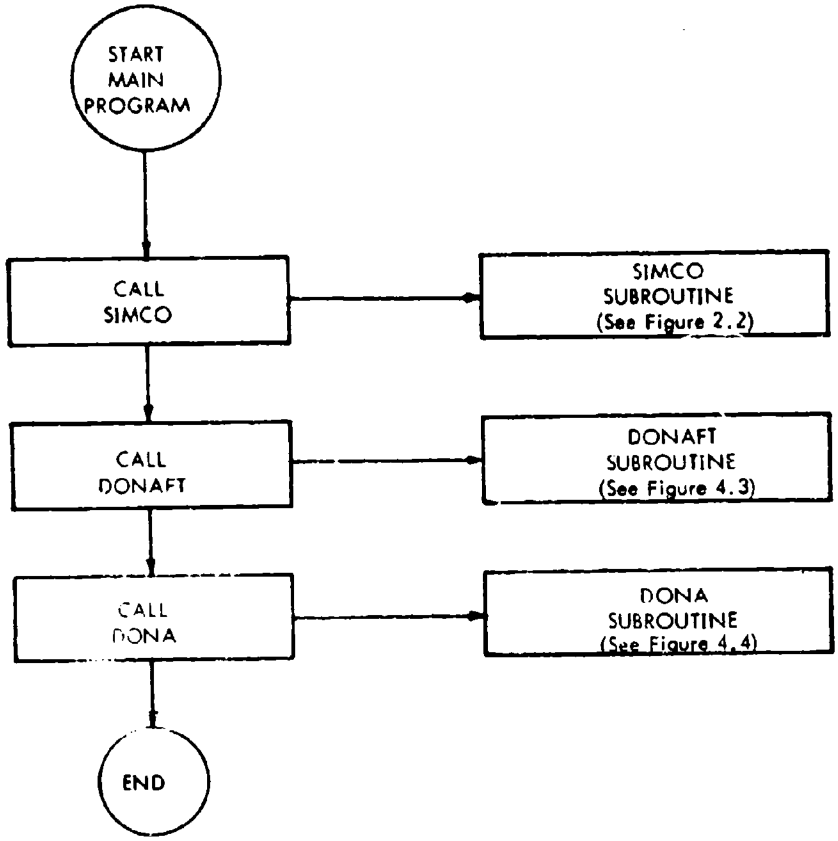


Figure 4.2 Laboratory System Computer Flow Diagram

plementary input of stock received. In this case, the wholesale cost was \$50 per unit and so the total wholesale cost was equal to the stock received times \$50. This equation was then put in the program. Having completed this input-output analysis, the remaining variables were examined for their appropriateness of describing the state of SIMCO. The remaining variables were subsequently identified and labeled as state variables. For the purposes of testing the laboratory system, all variables were not used. The variables selected for the checkout were 2 input, 2 output, and 5 state variables. The input variables were the actual demands during the week for product 1 and for product 2, (ADDW and ADDW1). The state variables were units ordered for products 1 and 2 (U0 and U01), actual inventory end of week for product 1 (AIEW), and forecasted demand for product 1 and 2 (FD and FD1). The output variables were salaries for personnel on product 1 (SAL) and wholesale cost of product 1 received from supplier (WC).

The analysis of the "real world" model was not completed with the identifying of the input, output and state variables. The DONA methodology requires a matrix inversion at a point in its sequence of operations. The elements in the matrix to be inverted were initially derived from a series of values from the "real world" inputs, outputs and state variables. A matrix inversion requires that the matrix be non-singular. Specifically, the matrix must not contain any rows that are linear combinations of another and there must not be a column or row with all zeroes for the values. The computer program for the laboratory showed that SIMCO was indeed producing both of these objectionable traits

and thereby causing matrix singularity. This says that SIMCO in simulating the real world was producing redundant information. Linear combinations of some variables were appearing in the SIMCO simulation. The other untenable situation, all zeroes in a row or column, resulted from the characteristics of the DONA methodology. The method to produce the DONA model requires that the variables not remain at a constant value or that the slope (first derivative) not be constant.

The point to be made in using SIMCO or using a real world organization is that a careful analysis must be performed in selecting the variables and noting the values taken by the variables. The computer program has several built-in features to detect the matrix singularities but the labeling of variables as input, output or state must be a manual task.

To continue the description of the laboratory system, the DONA methodology block in Figure 4.1 contains the multiple regression analysis procedure for developing the "S" matrix. In the laboratory system the "S" matrix will characterize the SIMCO Sales Company. The computer program for this segment of the laboratory system has some unique features some of which were dictated by the limitations of the computer (UNIVAC 1108) and some added to produce traceability and analysis of the computations used in the DONA methodology. These features may be recognized by referring to the computer flow diagram in Figure 4.3. The computer program for this segment of the laboratory system is called DONAFT (DONA Fit) and implements the DONA methodology.

The DONAFT subroutine begins by admitting the preselected input,

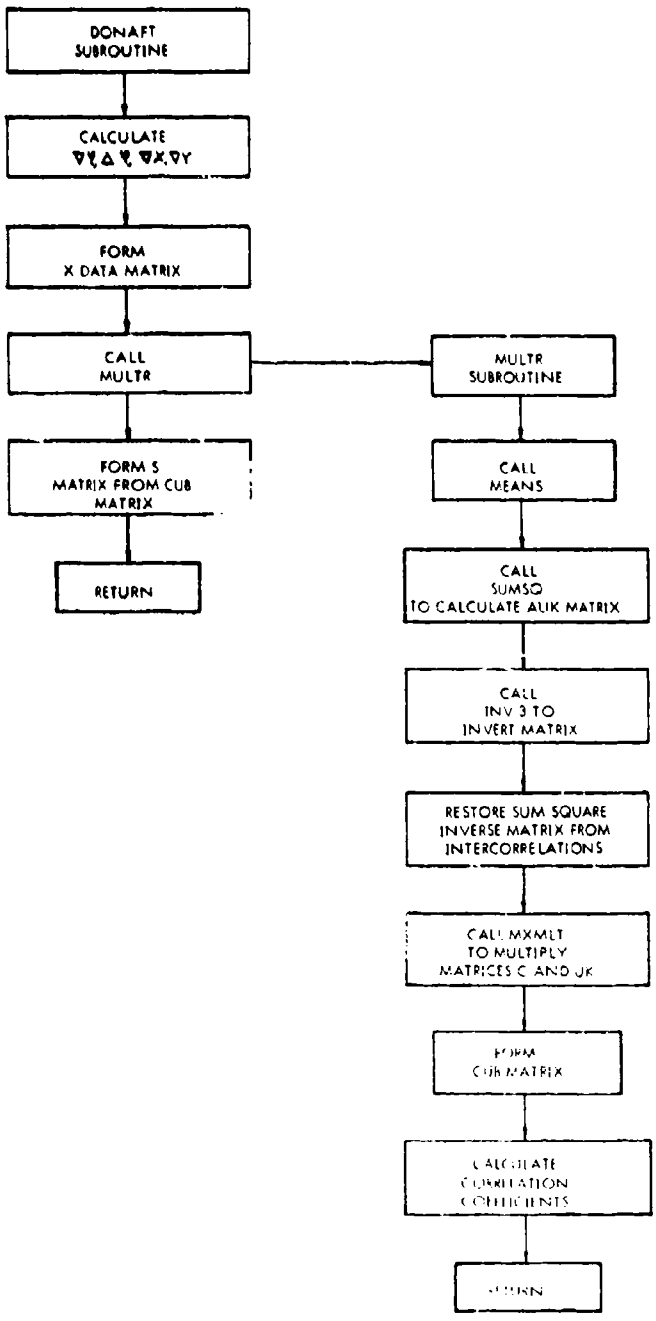


Figure 4.3. Flow Diagram of the DONAFT Subroutine

output and state variables of SIMCO generated for 82 weeks of SIMCO operations. The appropriate forward and backward differences were calculated for all observations (82 weeks). The forward and backward differences as defined in Chapter III were the difference in value between the present time and one time past for the backward difference and the difference between one time interval in the future and the current time for the forward difference. The backward differences of the input, output and state variables were determined and the forward differences of only the state variables were determined. These differences were performed at each time interval beginning with week 2 and ending in week 81. Weeks 1 and 82 had to be available because in the series of differences, the backward difference requires one interval in the past, week 1 when week 2 is considered. Similarly, the forward difference requires one interval in advance of the series, week 82 when week 81 is considered. The results of the differences gave [illegible] sets of differences for each of the 2 input, 2 output, and 5 state variables.

The array of differences must be formed for the next process in a manner such that the top half rows of the matrix contain the independent variables. Each column of the matrix represents a time interval. This matrix is called the data matrix or "X" matrix.

The "X" matrix is transferred into the MULTR (Multiple Regression) subroutine. The mean of each variable was determined and subtracted out leaving the deviation from the mean for each variable. The deviations from the mean are still in the difference form. The deviations do not reflect back

to the real value of the individual variable. All computations are based on the forward and backward differences. Having the deviations from the mean, the sum squares are computed for all combinations of the variables. The "A" matrix and "K" matrix as described in Appendix C, have now been determined.

The computer program produces these sum squares in a square array AUK (A union K). In actuality, the square A matrix is the upper left quarter of the square AUK matrix and the square K matrix is the upper right quarter of the AUK matrix. The usual procedure would be to take the A matrix into a matrix inversion subroutine to perform the inversion. However, one of the aforementioned problems now arise. Assuming the A matrix has had the zeroes removed, if they were present in a whole row or column, and assuming linear combinations are not present in the A matrix, the nature of the A matrix causes the inversion process to exceed the computer capability. Specifically, the A matrix contains values ranging from 10^9 to 10^{-6} when using the SIMCO data. Even if the program were written in double precision, it is doubtful that a good inversion could be obtained. The multiplication processes in the matrix inversion would exceed the capability of the computer registers. The answer to this problem was to "normalize" the AUK matrix. This can legitimately be done through conversion of the A matrix into a matrix of intercorrelations. The elements of the AUK matrix represent $n\sigma_{ij}$; i.e. the covariance times the number of sample points.

An element of the AUK matrix equals the sum of products $(x_i \cdot x_j)$. Then the covariance results from

$$\sum_{k=1}^{n} (x_{ik}\ x_{jk}) = n\sigma_{ij} \tag{4-1}$$

(origin at the means)

where σ_{ij} is defined as

$$\sigma_{ij} \triangleq \sum_{k=1}^{n} \frac{(X_{ik} - \overline{X}_i)(X_{jk} - \overline{X}_j)}{n} = \frac{\sum_{k=1}^{n} (x_{ik} \cdot x_{jk})}{n} \tag{4-2}$$

The ith element on the main diagonal (ith column and ith row) of the AUK matrix is $n\sigma_{ii}$, also written $n\sigma_i^2$. The same is true of the jth element; i.e., $n\sigma jj = n\sigma_j^2$. The AUK matrix has been written into the AAA matrix so that the AAA matrix may be used as a working matrix.

The intercorrelations or Pearson product-moment correlation coefficients are defined as

$$\rho_{ij} \triangleq \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j} \tag{4-3}$$

This equation may be used by modification as follows

$$\rho_{ij} = \frac{n\sigma_{ij}}{n\sigma_i \cdot \sigma_j} = \frac{n\sigma_{ij}}{\sqrt{n\sigma_i^2 \cdot n\sigma_j^2}} \tag{4-4}$$

By applying this relation to the AAA matrix the resulting matrix, the intercorrelation matrix, ranges in value from -1 to +1. The intercorrelation matrix was then applied to a matrix inversion subroutine which also had an internal matrix inversion accuracy improvement routine. The inverse is returned

to the form of sum of products through an inverse transformation using the inter-correlation relation (4-4) in reverse. The A inverse matrix, submatrix of the inverted AUK matrix, is pre-multiplied with the K matrix in conformance with the multivariate regression analysis (see Appendix C). The result is the CUB (C union B) matrix where the B matrix, sub-matrix of CUB, is the matrix of coefficients of regression (Appendix C).

This concludes the operation of MULTR subroutine except for a very helpful set of statements which compute and printout the statistical properties for each dependent variable as a function of the independent variables. One important printout is the amount of variability that has been explained in the dependent variable through the independent variables.

The DONA methodology and the DONAFT subroutine is completed with one more process. The "S" matrix as defined in Chapter III, equation (3 - 17) is obtained by taking the right half of the CUB matrix, the C sub-matrix, and transposing it. The resulting S matrix characterizes SIMCO Sales Company.

The DONA model block in Figure 4-1 represents the DONA model as discussed in Chapter III. The process in this stage is depicted by the following matrix equations

$$\begin{bmatrix} \dot{Y}(t+1) \\ \overline{Y}(t) \end{bmatrix} = \begin{bmatrix} S \end{bmatrix} \cdot \begin{bmatrix} \dot{X}(t) \\ \overline{X}(t) \end{bmatrix} \qquad (4\text{-}5)$$

The left side of the equation is the output from the DONA model.

The DONA subroutine of the laboratory system computer program (see Figure 4.4) performs the calculations of the DONA model using equation (4-5).

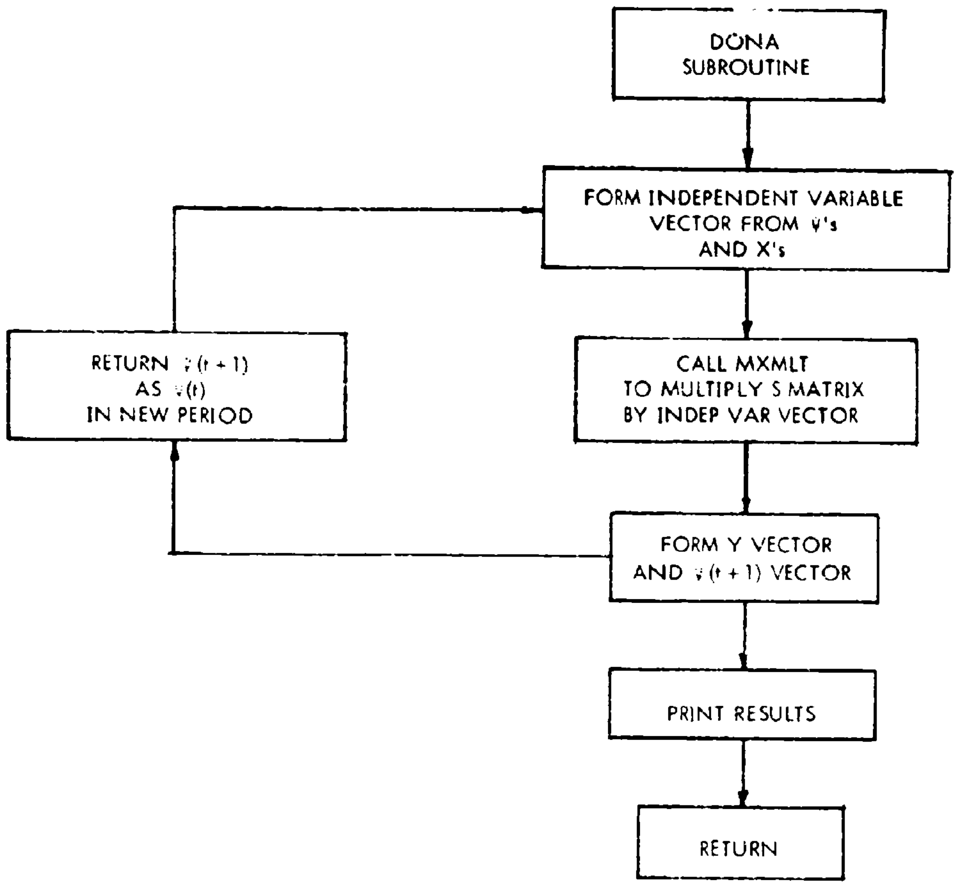


Figure 4.4 Flow Diagram of DONA Subroutine

The S matrix was developed in the DONAFT subroutine and entered in the DONA subroutine. The state variable initial conditions used for SIMCO are also imposed on the DONA subroutine for the first time interval only. After the first time interval the $\ddot{Y}(t + 1)$, produced as an output of the DONA model, are used as the input state variables for the next time interval. The X(t) inputs are provided the DONA model for each time interval. The outputs of the DONA model are the state variable vector for the future time interval and the output vector for the simulated system.

The outputs of SIMCO and DONA are finally compared as shown in Figure 4-1. The comparison is made on all parameters desired or deemed appropriate for consideration.

Operation of the Laboratory System

The operation of the computer-based laboratory system is one complete flow; however, the sequential actions may be portrayed as occuring in two major parts. Phase I is the initial phase with the purpose of determining the S matrix, the system characterization matrix. The flow through the laboratory system for this phase is shown in Figure 4.5 by the solid lines. In Phase I the data is taken from the actual system (SIMCO in this case) and entered into the DONAFT subroutine where the DONA methodology is accomplished. The result of this process is the S matrix.

Phase II in the operation of the laboratory system is the performance of the DONA model and comparison of the outputs. Having determined the S matrix

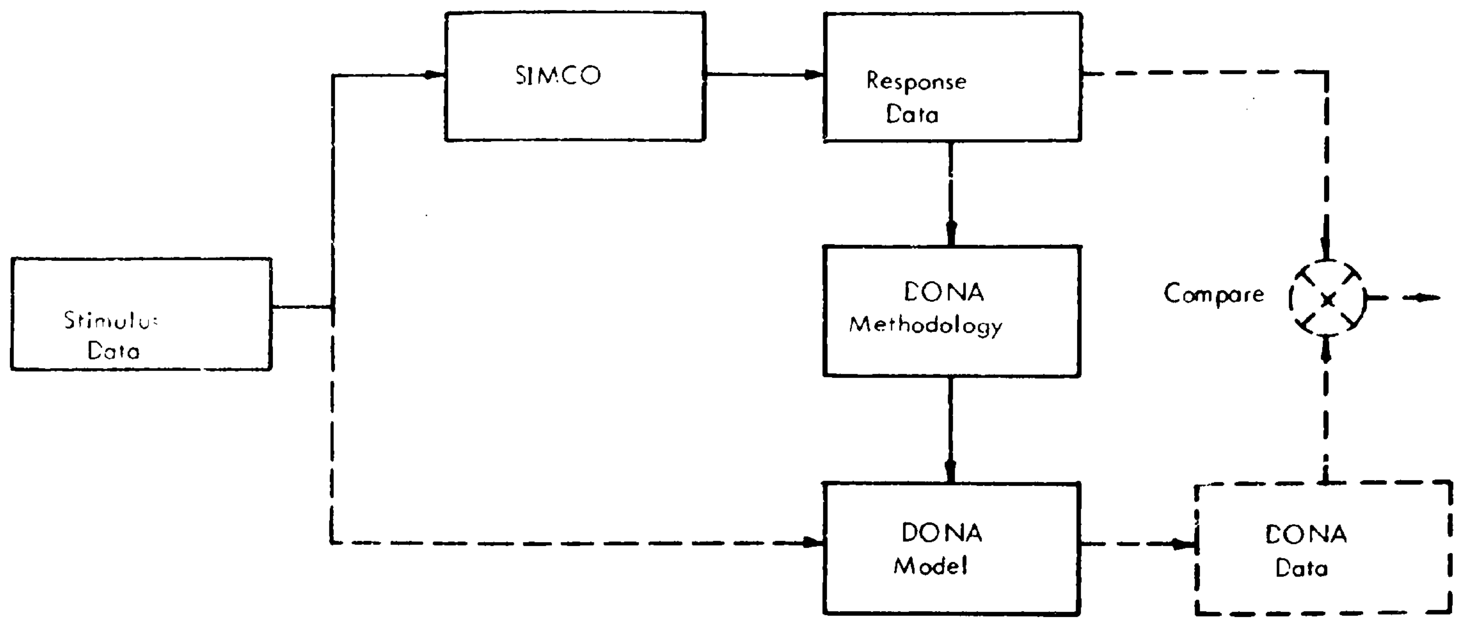


Figure 4.5 Phase I of the Laboratory System Operation

in Phase I, the DONA model is complete and ready to function. Phase II may be considered as the "free-running" state of the laboratory system in that both models are continuously producing outputs and state variables. This phase is shown in Figure 4.6 by the solid lines. It will be noted that the DONAFT subroutine has been "disconnected" in this phase of the laboratory operation.

The laboratory system has great flexibility primarily because it is a computer program and changes may easily be made. This flexibility may be illustrated in another way. Suppose that after having disconnected the DONA fit procedure the comparison procedure has signaled that the S matrix is no longer valid. That is, the S matrix no longer characterizes the system that it is representing. This requires generation of a new matrix or updating of the S matrix. The laboratory system perm s this updating in a very uncomplicated manner. The complete laboratory system is rerun using new data from the actual system to be modeled. The two phase operation occurs automatically and thus a new system matri; and its DONA model are generated.

Validation of the DONA Method

The major consideration in developing and using the laboratory concept was to have control over the range and types of variables which the DONA model may experience. With such exact control then some conclusions may be reached on the performance of the DONA model and the method used to produce the model.

The inputs or stimulus data to SIMCO and DONA may be any one or a

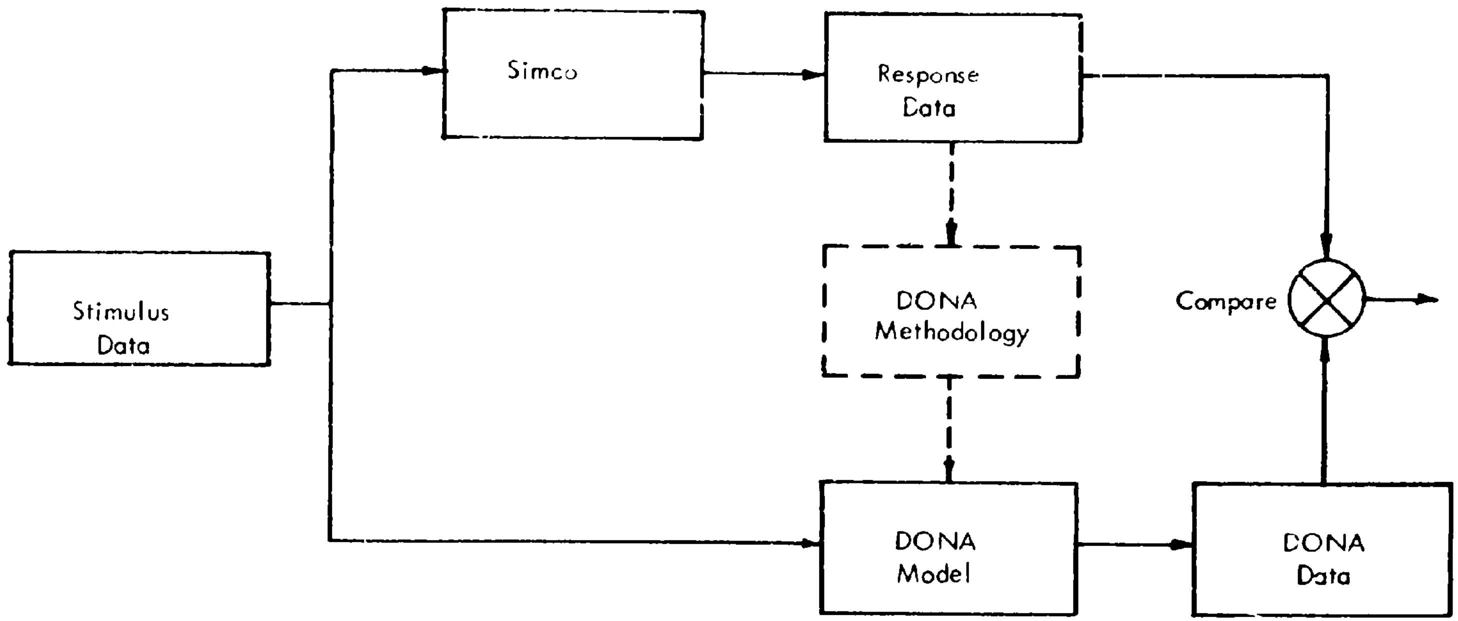


Figure 4.6 Phase II of the Laboratory System Operation

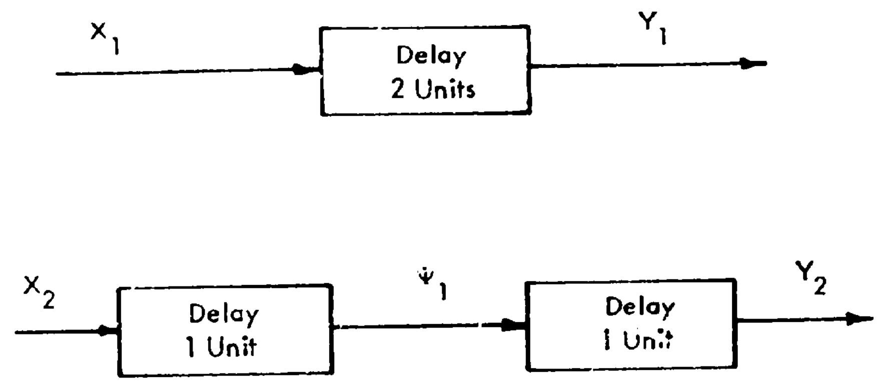
combination of signals composed of impulses, step functions, sinusoids, or random noise. In the early stages of conducting the experiment, only one type of stimulus was considered. It was a normally distributed "noise" with a mean of 50 and a standard deviation of 15. This stimulus was the customer demand as explained in Chapter II for the SIMCO simulation.

The comparison step in the laboratory system was based on computer printouts of the SIMCO and DONA state variables and output variables. In a line by line comparison of the SIMCO result and the DONA result much information was gained on the performance of the DONA · del.

Results of the Laboratory System

The computer program for the laboratory system in Figure 4.1 was written according to the flow diagrams in Figures 4.2, 4.3, and 4.4. The complete program listing is shown in Appendix D. The program was checked out by running the individual subroutines and the parts tested out successfully on the computer (UNIVAC 1108).

Two test models were developed to discover some basic behavior characteristics of the DONA model. The first test model in Figure 4.7 contained only delays. A discrete approximation of a sawtooth wave with a period of 8 time units was fed in to the model. In one experiment the sawtooth was delayed one time unit followed by a second delay of one time unit. A state variable was monitered between the two delays. The same sawtooth wave was also fed in to a two unit time delay. The above was repeated but used an input of noise



TEST PLAN

	X_1	X_2
Experiment	Noise	Saw tooth
Experiment	Saw tooth	Noise

FIGURE 4.7 FIRST TEST MODEL

rather than the sawtooth wave. The results were that a very high correlation (100%) could be obtained by using one time unit delay regardless of input--noise or sawtooth wave. The high correlation is only obtainable by using a state variable. When an output variable is used for correlation after one time unit delay, very poor results are obtained. An explanation for the better performance of a state variable over an output variable comes from the state equation model. The state variables have the advantage of knowing values one time interval in the past, whereas, the output variables are determined from current time calculations.

When a two unit time interval was encountered, poor correlation resulted with either input. This is because only a first time difference is used in the method. Another result from the model in Figure 4.7 was the relatively high correlation (30%) from the output of the second delay when noise was applied. In comparison, a sawtooth input resulted in only a 2% correlation at the output of the second delay. This implies that the higher frequency noise is less filtered by the two cascaded delays than the lower frequency sawtooth wave. The frequencies of the noise and sawtooth signals result from the first differences (in time) taken in the forward and backward differences of the variables, equations (3.19) and (3.20).

The second test model used to discover the basic characteristics of the DONA method is shown in Figure 4.8. The objective was to examine what happened when two signals were summed. The result was that there was 100% correlation of the output with the inputs. Further, the output was equally

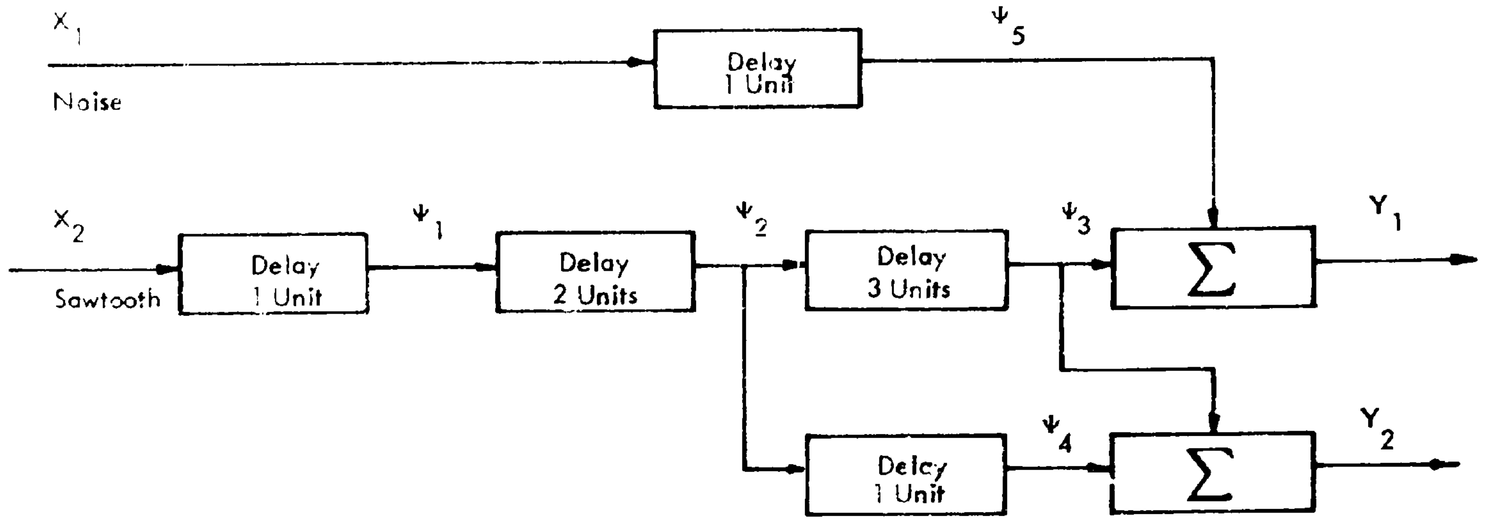


FIGURE 4.8 SECOND TEST MODEL

weighted and weighted to the maximum extent on each of the inputs. This high correlation in the summing operation supports the theory that the results of the output variables are highly significant when current time information is used rather than delayed or past time is used in deriving the output prediction.

The laboratory computer program was also successfully operated with the SIMCO simulation. The results of this experiment are shown in Appendix E. It was noted that the behavior of the SIMCO predictions was not good. It was expected that the SIMCO results would not be closely comparable because the DONA model indicates the low (1% to 58%) correlation of the variables. If additional analysis and adjustment of the DONA model were made to include more of the significant variables, higher correlation would result. This was done in the experiment and the correlation on one variable was raised from 23% to 58%. But there must also be some consideration given to the behavior of the DONA as revealed in the results of the test models. Therefore, the construction of the model is highly significant in the worth of its predictive behavior.

Developing the considerations to be employed in the construction of a model was a major result of this study. The DONA model characteristics dictate that in modeling the real-world system the total delays in the system must be represented in the DONA model by cascaded unit delays. The question is, how many of these delays are needed for the representation? This is answered through the comparison of the real-world and DONA values in the MULTR subroutine of the laboratory computer program. The MULTR subroutine shows where the DONA model needs improvement. It shows which variables are

accounting for the variability and also shows the strength of the variables' contribution.

Another consideration in model construction is the tendency of DONA to use "alibi" variables; that is, a variable completely unrelated to the operation or flow may be called in from another segment of the model to explain the variability of the variable under study. Again, the MULTR subroutine will show the effects of the "alibi" variables. It may be interesting in further study to attempt to force out these unwanted variables of the solution.

Model construction may be further enhanced through a technique to discover linear combinations. A problem mentioned earlier in this chapter was the occurrence of linear combinations in the characterization of a man-organized sysrem. When a matrix inversion was attempted, it was to no avail because of the singular matrix. Linear combinations in this connection are considered as the form where one equation is identical to the other multiplied by a constant. It is recognized that there are other forms of linear combinations.

During the course of this research, it was discovered that there was a way to recognize the troublesome linear combinations. Consider the following matrix as representing a linear combination of two equations.

$$\begin{bmatrix} x_{12} & x_{22} & x_{23} & x_{24} \\ ax_{12} & ax_{22} & ax_{23} & ax_{24} \end{bmatrix} \qquad (4\text{-}6)$$

Assume it is then placed in a larger matrix and made symmetrical.

$$\text{Linear combination} \quad \begin{bmatrix} x_{11} & x_{12} & ax_{12} & x_{14} \\ x_{12} & x_{22} & ax_{22} & x_{24} \\ ax_{12} & ax_{22} & a^2x_{22} & ax_{24} \\ x_{14} & x_{24} & ax_{24} & x_{44} \end{bmatrix} \tag{4-7}$$

Because of considerations discussed earlier, it was required to transform the matrix into intercorrelation form. Then by the definition of intercorrelation, equation (4-3), the values in the main diagonal will always be +1.0. The matrix (4-7) in intercorrelation form will have a main diagonal with all values of +1.0; i.e., $x_{11} = x_{22} = a^2x_{22} = x_{44} = 1.0$. The result is that $a^2 = 1.0$ and $a = \pm 1$. In matrix (4-7) ax_{22} always equals $\pm$ 1 since $x_{22} = 1.0$. The significance of this is that a linear combination of the type described may be detected in the intercorrelation matrix by noting the presence of an element equal to $\pm$ 1 not on the main diagonal. This condition must exist for a linear combination but it is not the only condition. The inspection for ones in the intercorrelation matrix is useful as a rapid means to determine if there is a possibility of linear combinations. The results of the laboratory system confirmed the above. In Appendix E the AUK matrix, AAA matrix, and Intercorrelation matrix are symmetrical in . upper left quarter. These parts of the matrices are the only parts used in the matrix inversion routine. As expected, the Intercorrelation matrix contains ones on the main diagonal and there are no other ones in the upper left quarter of the computer printout. It will also be noticed that the inverted matrix is symmetrical. In Appendix E the Improved Inverse Matrix A (Intercorrelated Form) and the Improved Inverse Matrix A

(No Intercorrelations) are symmetrical. If any of these matrices are unsymmetrical during the course of using the laboratory system, the input data is immediately suspected of containing irregularities.

It was the plan of research to understand some fundamental characteristics of the behavior of the DONA model and to understand how to apply the method. This was accomplished but there are many areas that need further exploration. Since this study was the first attempt to implement the DONA method, this has been only a beginning in the research required to adequately understand and apply the method. It appears that enough has been discovered in this initial effort to be at least familiar with the capabilities and limitations in the use of the DONA model.

CHAPTER V

SUMMARY

The Approach

This project resulted from a need to model a large governmental agency. The means of developing the model must be more timely than previous methods which may require up to 10 years for formulation. It was desired that the model predict in gross terms as opposed to detailed prediction of system variables.

The approach to this problem was to use the best features of modeling from the economist's view and the control theory standpoint. The state variable approach was used to accomplish those objectives. The use of state variables to produce a model was known as Dynamic Organizational Network Analysis (DONA). The DONA model for modeling a man-organized system was developed from a methodical means of evaluating the man-organized system. It is this method of producing the DONA model that is the unique feature of this project.

Since the DONA method of modeling is a new approach, a question arises concerning its validity. Therefore, it is desired to validate the DONA model and this in turn will validate the method used to produce the model. A computer-based laboratory system was conceived for validation purposes. The idea of a "laboratory" was used because of the required control of the stimulus data. Such control cannot be achieved through the use of the actual system. The role of the real world system in the laboratory concept was played by a computer simula-

tion of a sales distributor company. The SIMCO simulation was used because of its known characteristics. The outputs of SIMCO, considered as standard for comparison, were matched with the outputs of the DONA model. In this manner a judgment could be made on the performance of the DONA model and the method used to produce the model.

The Investigation

The investigation was centered around the feasibility of developing the approach and converting it into a suitable computer program. This portion of the project was indeed a huge undertaking. The DONA approach requires extensive matrix manipulations. The matrix manipulation would not be such a problem except for two very peculiar problems associated with modeling a large man-organized system. To appropriately characterize a large system, many variables need be considered. An example of this is the relatively simple SIMCO simulation which contains about 50 variables. If all these variables were considered, the DONA method dictates the mathematical operations of matrices with a size 100 by 100. The second problem that arises was one that was completely unforeseen in the early stages of development. It appears that in quantitatively characterizing a man-organized system many redundancies occur. This results in linear combinations appearing in the matrix descriptions of the system. The DONA method requires inversion of the previously described large matrices. The presence of linear combinations in a matrix to be inverted causes well known problems. This problem was resolved by removing the linear combina-

tions and thereby avoiding matrix singularities. A third problem which arose was due to computer capacity limitations. The system description matrices contain numbers ranging from 10^9 to 10^{-6}. The problem was solved by using a "normalized" form of the matrix. The matrix was converted to intercorrelations which range from +1 to -1. With the above major problems resolved the computer program was complete.

The investigation then proceeded into the validation phase. The input or stimulus data was conceived of consisting of one or a combination of impulses, step functions, sinusoids, and random noise. A comparison was made of the outputs of SIMCO and DONA model when the same stimulus was applied to each model.

Results

The primary objective of developing the DONA approach and converting it to a computer program was achieved. This program is shown in the flow chart in Figure 4.2. The computer listing for the program is in Appendix D.

The validation results are shown in the computer printouts in Appendix E. It will be noted that the DONA and SIMCO outputs do not closely compare. The reason for this is explained in an earlier portion of the same printout. The program contains a unique feature of evaluating the "framing" of the problem. It is important in the initial stages of modeling to carefully select the real world variables for analysis. It is highly unlikely that all variables may be entered into any model of a man-organized system. Therefore, a careful screening action

must be performed to select variables thought to be most influential in the performance of the system. The screening action that took place in the SIMCO simulation is a good illustration of the "framing" requirements. As stated earlier SIMCO contained about 50 variables. After analysis of the significance of each variable, 2 output, 2 input, and 5 state variables were identified for use in the modeling effort. Then the computer program evaluates the contribution of these variables toward explaining the variability of the model outputs. This accounting for the variability in the dependent variables (outputs) as explained by the variability of the independent variables (inputs) is reflected in the coefficients of correlation in the multiple regression analysis. The square of the coefficients may be considered as percentages with 100% meaning perfect correlation between dependent and independent variables and 0% indicating no correlation. The computer results show that the correlations range from 0.4% to 58%. Then the poor comparison of model outputs is to be expected. But what this is saying is that there have not been either enough variables entered in the model to characterize the system performance or the ones currently under consideration are not contributing enough to explain the system performance. An illustration of this is an experiment actually performed to try to increase the "accounting for variability." Prior to accomplishing the 58% that variable had given results of about 23% when another choice of quantities had been entered in the model. The 35% increase was due to dropping out one variable measured from SIMCO and substituting another one. It is felt that further work on this problem would yield better comparability.

Areas of Further Study

The concept of modeling through the state-space approach and the DONA methodology looks extremely promising. This thesis was a beginning in the development of the concept and implementation through a computer program. It is recognized that many areas have not been considered. It would be desirable to explore the basic characteristics of the DONA model and the method used to produce the model. Detailed experimentation in the laboratory system would provide some of these answers. The laboratory system was conceived to permit control of the variables to allow discovery of these basic characteristics. A systematic application of the above alluded to types of stimulus data would lend understanding to the responsiveness and frequency characteristics of the DONA model.

Further effort can be devoted to enriching the computer program to automatically screen the system data to detect the possibility of linear combinations. Since this condition causes an automatic "no solution" in the DONA methodology, this area should get priority consideration. It is believed that this problem can be easily overcome.

Also for further study is the area of comparison in the laboratory system. The comparison at present is based on a judgment of the comparability of two columns of figures. A statistical means should be adopted to permit a more critical judgment of the results. One possibility to be used in the criterion for agreement of outputs is the use of the quadratic form of the covariance matrix.[13]

This statistical procedure lends itself to an analysis involving multiple variables. Further information on this proposal is contained in a paper by B. B. Lukens and R. A. Brown entitled "Experimental Design Considerations in Validating a Method of Modeling a Man-Organized System."[14]

Exploration of these areas would add tremendous value to the DONA concept of modeling a large man-organized system. A significant achievement would be realized by applying this concept of modeling to the real world system.

APPENDIX A

COMPUTER PROGRAM FOR SIMCO

AND SIMCO OUTPUT

```
@ FOR SIMCO
C      PROGRAM SIMCO
       DIMENSION AR(5),PIPLIN(9),DDP(11),R(22),A(22)
       DIMENSION STDEV(22)
       COMMON/RAND/IRD,NSTART,RND1,IRN,NOPS
       NSTART=123456
       CALL INIT
     1 FORMAT(14H WEEKLY DEMAND,F27.4,F25.4/23H BACKORDER QUEUE LENGTH,F1
      18.4,F25.4/25H INVENTORY AT END OF WEEK,F16.4,F25.4/17H SUPPLY BACK
      2ORDER,F24.4,F25.4/20H ACCOUNTS RECEIVABLE,F21.4,F25.4/13H CASH ON
      3HAND,F28.4,F25.4/6H SALES,F35.4,F25.4/24H GROSS INCOME FROM SALES,
      4F17.4,F25.4/20H WEEKLY GROSS PROFIT,F21.4,F25.4/18H WEEKLY NET PRO
      5FIT,F23.4,F25.4/20H UNITS OF LOST SALES,F21.4,F25.4)
     2 FORMAT(//33X,8H AVERAGE,10X,19H STANDARD DEVIATION)
     3 FORMAT(///3X,4HWEEK,5H ADDW,5X,3HCBQ,5X,4HAIEW,5X,3HASB,7X,3HTAR,
      19X,3HCOH,5X,3HTUS,6X,3HGIS,7X,3HWGP,7X,3HWNP,4X,3HULS,5X,2HFD,3X,3
      2HPMD/)
       WRITE (6,3)
C INITIALIZE VALUES
C INITIALIZE DATA ACCUMULATION VALUES AT ZERO
       DO 4 K=1,22
     4 R(K) = 0.0
C INITIALIZE PIPELINE FOR THE NEXT 8 WEEKS
       DO 5 M=1,6
    5   PIPLIN(M)=50.0
       PIPLIN(7)=0.0
       PIPLIN(8)=0.0
       TMU = 50.0
       SD = 15.
       UO=50.0
       ULS = 0.0
       AIEW=150.0
       CBQ = 0.0
       DO 6 J = 1,10
     6 DDP(J)=50.0
       FD=50.0
       PMD = 0.0
       SSF = 0.0
C INITIALIZE ACCOUNTS RECEIVABLE FOR THE NEXT 4 WEEKS
       DO 7 K=1,4
     7 AR(K)=2500.0
       TAR=10000.0
       COH=4000.0
C RUN FOR 100 WEEKS
       DO 25 JWKS=1,100
       CALL RANDM(A,B)
C CALL SUPPLY TO DETERMINE LEAD TIME FOR UNITS ORDERED
       CALL SUPPLY(PIPLIN,SR,UO,H)
C FILL CUSTOMER BACKORDERS IF ANY
       AIBW = AIEW + SR
       IF (CBQ - AIBW) 8,8,9
```

```
    8 BS = CBQ
      AAIBW = AIBW - CBQ
      CBQ = 0.0
      ULS = 0.0
      GO TO 10
    9 BS = AIBW
      AAIBW = 0.0
      ULS = (CBQ - AIBW) * .20
      CBQ = (CBQ - AIBW) * .80
   10 N = CBQ
      CBQ = N
      N = ULS
      ULS = N
C CALL DEMAND TO DETERMINE DEMAND FOR THIS WEEK
      CALL DEMAND(ADDW,PMD,TMU,SD,SSF,A)
C DETERMINE TOTAL SALES DURING THE WEEK
      IF (ADDW - AAIBW) 11,11,12
   11 TUS = BS + ADDW
      GO TO 13
   12 TUS = BS + AAIBW
C DETERMINE BACKORDER QUEUE LENGTH
      CBQ = CBQ + (ADDW - AAIBW)
C DETERMINE GROSS INCOME,GROSS PROFIT, AND ENDING INVENTORY
   13 GIS = 100. * (1.0 - PMD) * TUS
      WGP = GIS - 50.0 * TUS
      AIEW = AAIBW - ADDW
      IF (AIEW) 14,15,15
   14 AIEW = 0.0
   15 CONTINUE
   16 FC=2000.
C DETERMINE CASH ON HAND
   17 COH = COH + AR(1) + .5 * GIS - SR * 50. - FC
     1-(.2/52.) * 50. * AIBW - (.08/52.) * TAR
C UPDATE ACCOUNTS RECEIVABLE AND DETERMINE TOTAL A.R.
      DO 18 J = 1,3
   18 AR(J) = AR(J+1)
      AR(4) = .5 * GIS
      TAR = AR(1) + AR(2) + AR(3) + AR(4)
C DETERMINE WEEKLY NET PROFIT
      WNP = WGP - (.2/52.)*50. * AIBW - (.08/52.) * TAR - FC
      IF (ADDW -.70 * FD) 19,21,21
C FORECAST DEMAND FOR THE COMING WEEK AND DETERMINE IF A PRICE MARKDOWN/
C IS NECESSARY
   19 CALL FORCAS(DDP,FD,ADDW)
      IF (AIEW + PIPLIN(1) - 1.5 * FD) 22,22,20
   20 PMD = 0.15
      SSF = 0.05
      GO TO 23
   21 CALL FORCAS (DDP,FD,ADDW)
   22 PMD = 0.0
      SSF = 0.0
   23 CONTINUE
C CALL ORDER TO DETERMINE THE NUMBER OF UNITS TO BE ORDERED
      CALL ORDER (PIPLIN,UO,TUS,AIEW,FD,ASB,DI,OSB)
      WRITE (6,24) JWKS,ADDW,CBQ,AIEW,ASB,TAR,COH,TUS,GIS,WGP,WNP,ULS,FD
     *,PMD
   24 FORMAT(I7,F6.0,F8.2,2F9.2,F11.2,F12.2,F6.0,3F10.2,F6.1,F6.0,F5.2)
C ACCUMULATE SIGNIFICANT DATA
      R(1) = R(1) + ADDW
```

```
      R(2) = R(2) + ADDW**2
      R(3) = R(3) + CBQ
      R(4) = R(4) + CBQ**2
      R(5) = R(5) + AIEW
      R(6) = R(6) + AIEW**2
      R(7) = R(7) + ASB
      R(8) = R(8) + ASB**2
      R(9) = R(9) + TAR
      R(10) = R(10) + TAR**2
      R(11) = R(11) + COH
      R(12) = R(12) + COH**2
      R(13) = R(13) + TUS
      R(14) = R(14) + TUS**2
      R(15) = R(15) + GIS
      R(16) = R(16) + GIS**2
      R(17) = R(17) + WGP
      R(18) = R(18) + WGP**2
      R(19) = R(19) + WNP
      R(20) = R(20) + WNP**2
      R(21) = R(21) + ULS
   25 R(22) = R(22) + ULS**2
C IF WEEK IS LESS THAN 100, RETURN TO THE TOP OF THE LOOP
C WHEN WEEK EQUALS 100, CONTINUE
C DETERMINE AVERAGES AND VARIANCES FOR ALL SIGNIFICANT DATA
      DO 26 I=1,21,2
      A(I)=R(I)/100.
   26 STDEV(I)=SQRT((R(I+1)-R(I)**2/100.)/99.)
C   PRINT COLUMN HEADINGS
      WRITE (6,2)
C   PRINT RESULTS
      WRITE (6,1) (A(I),STDEV(I),I=1,21,2)
      STOP
      END
C FOR DEMAND
      SUBROUTINE DEMAND(ADDW,PMD,TMU,SD,SSF,V)
      D = TMU + SD * V
      IF (D) 1,2,2
    1 D = 0.0
    2 IF (PMD - 0.15) 3,4,4
    3 SSF = 0.0
      GO TO 5
    4 SSF = 0.05
    5 ADDW = D * (1.0 + SSF)
      N = ADDW
      ADDW = N
      RETURN
      END
C FOR SUPPLY
      SUBROUTINE SUPPLY (PIPLIN,SR,UO,X)
      DIMENSION PIPLIN(9)
C   DETERMINE SHIPMENTS RECEIVED
      SR = PIPLIN(1)
C   UPDATE SUPPLY
      DO 2 K=1,7
    2 PIPLIN(K) = PIPLIN(K+1)
      PIPLIN(8) = 0.0
C   DETERMINE LEAD TIME
      M=X*100.
      IF (M-9) 3,3,4
```

```
  3 LT = 4
    GO TO 11
  4 IF (M-24) 5,5,6
  5 LT = 5
    GO TO 11
  6 IF (M-74) 7,7,8
  7 LT = 6
    GO TO 11
  8 IF (M-89) 9,9,10
  9 LT = 7
    GO TO 11
 10 LT = 8
C   INCREASE APPROPRIATE WEEKS SUPPLY ORDERED BY UNITS ORDERED THIS WEEK
 11 PIPLIN(LT) = PIPLIN(LT) + UO
    RETURN
    END
* FOR FORCAS
      SUBROUTINE FORCAS(DDP,FD,ADDW)
      DIMENSION DDP(11)
C   UPDATE DEMAND FOR THE LAST TEN WEEKS
      DO 1 IB=1,9
      L = 11 - IB
    1 DDP(L) = DDP(L-1)
      DDP(1) = ADDW
C   CALCULATE FORECASTED DEMAND
      FD=(DDP(1)+DDP(2)+DDP(3))/3.0
      RETURN
      END
* FOR ORDER
      SUBROUTINE ORDER(PIPLIN,UO,TUS,AIEW,FD,ASB,DI,DSB)
      DIMENSION PIPLIN(9)
C   CALCULATE DESIRED INVENTORY
      DI = 3.0 * FD
C CALCULATE SUPPLY BACKORDER
      ASB = 0.0
      DO 1 N = 1,8
    1 ASB = ASB + PIPLIN(N)
C CALCULATE DESIRED SUPPLY BACKORDER
      DSB = 6.0 * FD
      UO = TUS + .5 + (DI - AIEW + DSB - ASB)
      N=UO
      UO=N
      IF (UO) 2,3,3
    2 UO = 0.0
    3 CONTINUE
      RETURN
      END
* FOR RANOM,RANOM
      SUBROUTINE RANOM(DEVIOT,RND2)
      COMMON/RANO/IRD,NSTART,RND1,IRN,NOPS
      IRN=IABS(IRD**2+NOPS**2)
      IRD=(IRN-(IRN/1000000000)*1000000000)/1000
      IF(IRD.EQ.0)IRD=1
      NOPS=NOPS+1
      RND2=FLOAT(IRD)*0.000001
      DEVIOT=SQRT(-2.*ALOG(RND1))*COS(6.2831854*RND2)
      RND1=RND2
      RETURN
      END
* FOR INIT,INIT
      SUBROUTINE INIT
      COMMON/RANO/IRD,NSTART,RND1,IRN,NOPS
      IRD=NSTART
      RND1=FLOAT(NSTART)*0.000001
      NOPS=47436
      RETURN
      END
* XQT SIMCO
* FIN
* FIN
```

WEEK	ADDR	CB[illegible]	AIFB	ASB	TAB	[illegible]	TUS	SIS	[illegible]GP	WNP	ULS	FD	PM[illegible]
1	19.	.00	181.00	300.00	4450.00	2394.15	19.	1900.00	950.00	-1[illegible]01.46	.0	40.	.15
2	71.	.00	160.00	250.00	4967.50	3454.21	71.	6035.00	2485.00	428.70	.0	47.	.00
3	50.	.00	160.00	281.00	8967.50	4322.05	50.	5000.00	2500.00	445.42	.0	47.	.00
4	42.	.00	168.00	240.00	8567.50	4351.87	42.	4200.00	2100.00	46.43	.0	54.	.00
5	55.	.00	163.00	313.00	10367.50	3492.77	55.	5500.00	2750.00	492.13	.0	49.	.00
6	40.	.00	173.00	283.00	4350.00	3953.34	40.	4000.00	2000.00	-59.35	.0	46.	.00
7	37.	.00	184.00	233.00	4700.00	3744.08	37.	3700.00	1850.00	-704.27	.0	44.	.00
8	39.	.00	147.00	247.00	8550.00	5744.43	39.	3900.00	1950.00	-98.92	.0	39.	.00
9	38.	.00	109.00	247.00	7700.00	4345.51	38.	3800.00	1900.00	-140.12	.0	34.	.00
10	44.	.00	278.00	58.00	7900.00	-168.26	44.	4400.00	2200.00	125.92	.0	40.	.00
11	45.	.00	233.00	129.00	8300.00	1864.12	45.	4500.00	2250.00	183.77	.0	42.	.00
12	50.	.00	203.00	173.00	8850.00	3254.70	50.	5000.00	2500.00	437.73	.0	46.	.00
13	48.	.00	155.00	264.00	4350.00	5502.05	48.	4800.00	2400.00	344.58	.0	48.	.00
14	60.	.00	104.00	308.00	10150.00	7955.16	60.	6000.00	3000.00	951.88	.0	53.	.00
15	47.	.00	62.00	425.00	10250.00	12518.59	47.	4700.00	2350.00	313.27	.0	52.	.00
16	59.	.00	27.00	426.00	10700.00	12736.28	59.	5900.00	2950.00	917.00	.0	55.	.00
17	57.	.00	105.00	395.00	11150.00	9108.66	57.	5700.00	2850.00	801.69	.0	54.	.00
18	47.	.00	58.00	441.00	10500.00	12501.32	47.	4700.00	2350.00	313.65	.0	54.	.00
19	63.	.00	86.00	387.00	11300.00	11436.51	63.	6300.00	3150.00	1103.94	.0	56.	.00
20	52.	.00	34.00	478.00	10950.00	14922.59	52.	5200.00	2600.00	564.62	.0	54.	.00
21	45.	.00	106.00	387.00	10350.00	12126.70	45.	4500.00	2250.00	205.04	.0	53.	.00
22	50.	.00	114.00	361.00	10500.00	17024.24	50.	5000.00	2500.00	452.31	.0	49.	.00
23	65.	.00	74.00	352.00	10600.00	15134.35	65.	6500.00	3250.00	1204.96	.0	53.	.00
24	55.	.00	65.00	425.00	10750.00	16144.97	55.	5500.00	2750.00	710.38	.0	57.	.00
25	39.	.00	130.00	396.00	10450.00	13397.93	39.	3900.00	1950.00	-98.58	.0	53.	.15
26	55.	.00	75.00	396.00	10287.50	15494.36	55.	4675.00	1925.00	-115.83	.0	50.	.00
27	61.	.00	109.00	332.00	10087.50	15395.84	61.	4100.00	3050.00	1001.79	.0	52.	.00
28	43.	.00	173.00	310.00	9987.50	12988.74	43.	4300.00	2150.00	93.87	.0	53.	.00
29	37.	.00	136.00	347.00	9387.50	14640.91	37.	3700.00	1850.00	-197.71	.0	47.	.15
30	67.	.00	69.00	347.00	9997.50	17785.32	67.	5695.00	2345.00	303.62	.0	49.	.00
31	57.	.00	206.00	295.00	9697.50	11919.51	57.	5700.00	2850.00	784.50	.0	58.	.00
32	8.	.00	198.00	334.00	7997.50	12[illegible]4.78	8.	800.00	400.00	-1451.84	.0	44.	.15
33	40.	.00	150.00	334.00	7797.50	13014.67	40.	3400.00	1900.00	-650.07	.0	35.	.00
34	38.	.00	236.00	218.00	4850.00	10797.44	38.	3800.00	1900.00	-163.23	.0	29.	.00
35	54.	.00	311.00	89.00	4700.00	7416.76	54.	5400.00	2700.00	619.50	.0	44.	.00
36	76.	.00	235.00	139.00	10100.00	9946.69	76.	7600.00	3800.00	1724.65	.0	56.	.00
37	48.	.00	187.00	345.00	10800.00	11945.91	48.	4800.00	2400.00	338.19	.0	59.	.00
38	41.	.00	235.00	306.00	10950.00	9416.22	41.	4100.00	2050.00	-19.92	.0	55.	.15
39	29.	.00	226.00	346.00	9442.50	11286.48	29.	2465.00	1015.00	-1044.78	.0	39.	.15
40	63.	.00	193.00	306.00	8360.00	15709.98	63.	5355.00	2205.00	152.52	.0	44.	.00
41	78.	.00	65.00	314.00	9860.00	19969.61	78.	7800.00	3900.00	1857.33	.0	57.	.00
42	80.	15.00	.00	523.00	11060.00	23241.95	65.	6500.00	3250.00	1220.48	.0	74.	.00
43	18.	.00	173.00	522.00	11477.50	13767.91	33.	3300.00	1650.00	-407.27	.0	59.	.15
44	55.	.00	218.00	422.00	11137.50	11712.66	55.	4475.00	1925.00	-144.63	.0	51.	.00
45	39.	.00	192.00	409.00	9187.50	14451.10	39.	3900.00	1950.00	-108.56	.0	37.	.00
46	46.	.00	186.00	409.00	8737.50	18350.09	46.	4400.00	2300.00	750.90	.0	47.	.00
47	37.	.00	313.00	205.00	8437.50	9570.06	37.	3700.00	1850.00	-230.29	.0	41.	.00
48	44.	.00	269.00	205.00	8300.00	12334.39	44.	4900.00	2200.00	127.09	.0	42.	.00
49	67.	.00	407.00	.00	4700.00	4980.47	67.	6700.00	3350.00	1243.92	.0	44.	.00
50	45.	.00	362.00	104.00	9650.00	7437.27	45.	4500.00	2250.00	156.88	.0	52.	.00
51	60.	.00	302.00	151.00	10070.00	10202.81	60.	6000.00	3000.00	913.77	.0	57.	.00
52	81.	.00	221.00	274.00	12650.00	18378.12	81.	8100.00	4050.00	1972.96	.0	62.	[illegible]
53	29.	.00	197.00	418.00	12500.00	14664.10	29.	2920.00	1230.00	-858.65	.0	55.	.15

54	48.	.00	149.00	418.00	10290.00	19132.12	48.	3540.00	1460.00	-373.72	.0	51.	.00
55	61.	.00	98.00	418.00	10390.00	23107.64	61.	4100.00	3050.00	1005.49	.0	44.	.00
56	25.	.00	337.00	144.00	7540.00	12622.11	25.	2500.00	1250.00	-831.22	.0	45.	.15
57	62.	.00	275.00	144.00	9975.00	14380.70	62.	5270.00	2170.00	91.38	.0	49.	.00
58	50.	.00	225.00	231.00	9435.00	14454.01	50.	5000.00	2500.00	432.60	.0	46.	.00
59	60.	.00	309.00	92.00	9385.00	13618.59	60.	6000.00	3000.00	914.60	.0	57.	.00
60	44.	.00	265.00	267.00	10335.00	14994.67	44.	4400.00	2200.00	129.68	.0	51.	.00
61	64.	.00	201.00	267.00	10900.00	18762.81	64.	6400.00	3200.00	1132.27	.0	56.	.00
62	54.	.00	147.00	347.00	11100.00	21937.39	54.	5400.00	2700.00	644.27	.0	54.	.00
63	51.	.00	96.00	393.00	10450.00	25412.59	51.	5100.00	2550.00	505.35	.0	56.	.00
64	55.	.00	41.00	462.00	11200.00	28327.23	55.	5500.00	2750.00	714.31	.0	53.	.00
65	42.	.00	86.00	407.00	10100.00	27235.35	42.	4200.00	2100.00	59.85	.0	49.	.00
66	48.	.00	213.00	232.00	9800.00	21519.62	48.	4800.00	2400.00	339.73	.0	48.	.00
67	56.	.00	162.00	265.00	10050.00	24562.62	56.	5600.00	2800.00	742.62	.0	49.	.00
68	24.	.00	138.00	332.00	9500.00	24966.01	24.	2400.00	1200.00	-844.23	.0	43.	.15
69	60.	.00	178.00	232.00	8950.00	24057.16	60.	5100.00	2100.00	40.46	.0	47.	.00
70	56.	.00	148.00	276.00	9350.00	25904.16	56.	5600.00	2800.00	746.38	.0	47.	.00
71	45.	.00	204.00	227.00	8800.00	23441.49	45.	4500.00	2250.00	188.58	.0	54.	.00
72	56.	.00	186.00	286.00	10400.00	23981.82	56.	5600.00	2800.00	737.46	.0	52.	.00
73	76.	.00	110.00	391.00	11650.00	28140.35	76.	7400.00	3800.00	1794.31	.0	59.	.00
74	47.	.00	130.00	430.00	11200.00	27924.09	47.	4700.00	2350.00	298.73	.0	60.	.00
75	69.	.00	61.00	454.00	12400.00	31585.86	69.	6900.00	3450.00	1405.92	.0	64.	.00
76	44.	.00	17.00	589.00	11800.00	34555.05	44.	4400.00	2200.00	170.12	.0	53.	.15
77	37.	.00	199.00	365.00	9572.50	26919.01	37.	3195.00	1295.00	-765.11	.0	50.	.15
78	31.	.00	168.00	365.00	8590.00	28528.51	31.	2635.00	1085.00	-966.41	.0	37.	.15
79	71.	.00	97.00	365.00	8107.50	32950.57	71.	6035.00	2485.00	440.22	.0	46.	.00
80	69.	.00	189.00	235.00	9357.50	28739.44	69.	6900.00	3450.00	1386.95	.0	57.	.00
81	49.	.00	190.00	343.00	10235.00	27951.58	49.	4900.00	2450.00	388.29	.0	63.	.00
82	38.	.00	282.00	296.00	10817.50	22591.80	38.	3800.00	1900.00	-178.18	.0	52.	.15
83	61.	.00	285.00	272.00	10392.50	24926.31	61.	5185.00	2135.00	60.17	.0	49.	.00
84	50.	.00	195.00	272.00	9442.50	28413.21	50.	5000.00	2500.00	438.36	.0	50.	.00
85	38.	.00	157.00	302.00	8892.50	31111.18	38.	3800.00	1900.00	-151.18	.0	50.	.00
86	65.	.00	92.00	324.00	10292.50	34217.31	65.	6500.00	3250.00	1204.05	.0	51.	.00
87	47.	.00	234.00	243.00	10000.00	27640.01	47.	4700.00	2350.00	280.58	.0	50.	.00
88	58.	.00	259.00	180.00	10400.00	26413.67	58.	5800.00	2900.00	823.09	.0	57.	.00
89	46.	.00	213.00	309.00	10800.00	28947.86	46.	4600.00	2300.00	233.58	.0	50.	.00
90	73.	.00	140.00	309.00	11200.00	33790.28	73.	7300.00	3650.00	1591.81	.0	59.	.00
91	14.	.00	156.00	434.00	9550.00	33290.36	14.	1400.00	700.00	-1347.38	.0	44.	.15
92	59.	.00	97.00	434.00	9157.50	36653.17	59.	5015.00	2065.00	20.91	.0	49.	.00
93	59.	.00	162.00	310.00	9807.50	33646.58	59.	5900.00	2950.00	892.91	.0	44.	.00
94	65.	.00	123.00	284.00	9407.50	37195.39	65.	6500.00	3250.00	1199.37	.0	61.	.00
95	76.	.00	176.00	362.00	12507.50	33142.90	76.	7600.00	3800.00	1732.30	.0	67.	.00
96	26.	.00	150.00	500.00	11300.00	34932.82	26.	2600.00	1300.00	-751.23	.0	56.	.15
97	46.	.00	259.00	345.00	10305.00	30015.79	46.	3910.00	1610.00	-469.51	.0	49.	.00
98	56.	.00	203.00	395.00	9855.00	34300.12	56.	5600.00	2800.00	735.03	.0	43.	.00
99	68.	.00	342.00	139.00	9455.00	28754.11	68.	6800.00	3400.00	1304.61	.0	57.	.00
100	29.	.00	313.00	234.00	9605.00	29425.79	29.	2900.00	1450.00	-630.55	.0	51.	.15

	AVERAGE	STANDARD DEVIATION
WEEKLY DEMAND	50.3600	19.0320
BACKORDER QUEUE LENGTH	.1500	1.5000
INVENTORY AT END OF WEEK	172.1000	80.6899
SUPPLY BACKORDER	309.7500	107.3906
ACCOUNTS RECEIVABLE	9426.0000	1174.3795
CASH IN HAND	19319.1010	9898.6256
SALES	50.3600	19.3996
GROSS INCOME FROM SALES	4907.7500	1919.1147

WEEKLY GROSS PROFIT	2389.7590	725.6139
WEEKLY NET PROFIT	331.8138	729.8209
UNITS OF LOST SALES	.0000	.0000

STOP

APPENDIX B

MODIFIED SIMCO

COMPUTER LISTING

```
C FOR SIMCO
C       PROGRAM SIMCO
        DIMENSION AR(5),AR1(5),PIPLIN(9),PIPLIE(9),DDP(11),DDP1(11),Y(101)
       1,Y1(101),XM(100),XM1(100)
        COMMON/RANO/IRD,NSTART,RND1,IRN,NOPS
        NSTART=123456
        CALL INIT
      3 FORMAT (///3X,4HWEEK,4HADDW,4X,5HADDW1,6X,3HSAL,6X,3HFD1,7X,4HTUS1
       *, 6X,3HCOH,5X,3HTUS,6X,3HGIS,7X,3HWGP,7X,3HWNP,4X,3HULS,5X,2HFD,3X
       *,2HZT,3X,1HZ,3X,2HMT,4X,3H SH/)
        WRITE(6,3)
C INITIALIZE VALUES
C INITIALIZE PIPELINE FOR THE NEXT 8 WEEKS
        DO 5 M=1,6
    5    PIPLIN(M)=50.0
        DO 50 M=1,6
   50 PIPLIE(M)=50.0
        PIPLIN(7)=0.0
        PIPLIE(7)=0.0
        PIPLIN(8)=0.0
        PIPLIE(8)=0.0
        TMU = 50.0
        TMU1=50.0
        SD=20.0
        SD1=15.0
        UO=50.0
        UO1=50.0
        ULS = 0.0
        ULS1=0.0
        AIEW=150.0
        AIEW1=150.0
        COU = 0.0
        COU1=0.0
        DO 6 J = 1,10
    6 DDP(J)=50.0
        DO 60 J=1,10
   60 DDP1(J)=50.0
        FD=50.0
        FD1=50.0
        PMU = 0.0
        PMU1=0.0
        SSF = 0.0
        SSF1=0.0
C INITIALIZE ACCOUNTS RECEIVABLE FOR THE NEXT 4 WEEKS
        DO 7 K=1,4
      7 AR(K)=2500.0
        DO 70 K=1,4
   70 AR1(K)=2500.0
        TAR=10000.0
        TAR1=10000.0
        COH=4000.0
        COH1=4000.0
C INITIALIZE PERSONNEL ACTIONS
        SUMM = 0.0
        SUMM1=0.0
        Y(1)=3.0
        Y1(1)=3.0
        XM(1)=0.0
        XM1(1)=0.0
```

```
C RUN FOR 100 WEEKS
      DO 25 JWKS=1,100
      CALL RANDM(A,B)
C CALL SUPPLY TO DETERMINE LEAD TIME FOR UNITS ORDERED
      CALL SUPPLY(PIPLIN,SR,UQ,B)
C FILL CUSTOMER BACKORDERS IF ANY
      AIBW = AIEW + SR
      IF (CBQ - AIBW) 8,8,9
    8 BS = CBQ
      AAIBW = AIBW - CBQ
      CBQ = 0.0
      ULS = 0.0
      GO TO 10
    9 BS = AIBW
      AAIBW = 0.0
      ULS = (CBQ - AIBW) * .20
      CBQ = (CBQ - AIBW) * .80
   10 N = CBQ
      CBQ = N
      N = ULS
      ULS = N
C CALL DEMAND TO DETERMINE DEMAND FOR THIS WEEK
      CALL DEMAND(ADDW,PMD,TMU,SD,SSF,A)
C DETERMINE TOTAL SALES DURING THE WEEK
      IF (ADDW - AAIBW) 11,11,12
   11 TUS = BS + ADDW
      GO TO 13
   12 TUS = BS + AAIBW
C DETERMINE BACKORDER QUEUE LENGTH
      CBQ = CBQ + (ADDW - AAIBW)
C DETERMINE GROSS INCOME,GROSS PROFIT, AND ENDING INVENTORY
   13 GIS = 100. * (1.0 - PMD) * TUS
      WGP = GIS - 50.0 * TUS
      AIEW = AAIBW - ADDW
      IF (AIEW) 14,15,15
   14 AIEW = 0.0
   15 CONTINUE
   16 FC=777.0
C DETERMINE CASH ON HAND
   17 COH=COH+AR(1)+.5*GIS-SR*50.-(.2/52.)*50.*AIBW-(.08/52.)*TAR
C UPDATE ACCOUNTS RECEIVABLE AND DETERMINE TOTAL A.R.
      DO 18 J = 1,3
   18 AR(J) = AR(J+1)
      AR(4) = .5 * GIS
      TAR = AR(1) + AR(2) + AR(3) + AR(4)
C DETERMINE WEEKLY NET PROFIT
      WNP = WGP - (.2/52.)*50. * AIBW - (.08/52.) * TAR
      IF (ADDW -.70 * FD) 19,21,21
C FORECAST DEMAND FOR THE COMING WEEK AND DETERMINE IF A PRICE MARKDOWN/
C IS NECESSARY
   19 CALL FORCAS(DDP,FD,ADDW)
      IF (AIEW + PIPLIN(1) - 1.5 * FD) 22,22,20
   20 PMD = 0.15
      SSF = 0.05
      GO TO 23
   21 CALL FORCAS (DDP,FD,ADDW)
   22 PMD = 0.0
      SSF = 0.0
   23 CONTINUE
```

```
C CALL ORDER TO DETERMINE THE NUMBER OF UNITS TO BE ORDERED
      CALL ORDER (PIPLIN,UO,TUS,AIEW,FO,ASB,DI,DSB)
C END OF PRODUCT 1, BEGIN PRODUCT 2
      CALL RANDM(A,B)
      CALL SUPPLI(PIPLIE,SR1,UO1,B)
      AIBW1=AIEW1+SR1
      IF(CBO1-AIBW1) 80,80,90
   80 BS1=CBO1
      AAIBW1=AIBW1-CBO1
      CBO1=0.0
      ULS1=0.0
      GO TO 100
   90 BS1=AIBW1
      AAIBW1=0.0
      ULS1=(CBO1-AIBW1)*.20
      CBO1=(CBO1-AIBW1)*.80
  100 N=CBO1
      CBO1=N
      N=ULS1
      ULS1=N
      CALL DEMND  (ADDW1,PMD1,TMU1,SD1,SSF1,A)
      IF (ADDW1-AAIBW1) 110,110,120
  110 TUS1=BS1+ADDW1
      GO TO 130
  120 TUS1=BS1+AAIBW1
      CBO1=CBO1+(ADDW1-AAIBW1)
  130 GIS1=100.*(1.0-PMD1)*TUS1
      WGP1=GIS1-50.0*TUS1
      AIEW1=AAIBW1-ADDW1
      IF (AIEW1) 140,150,150
  140 AIEW1=0.0
  150 CONTINUE
  160 FC1=777.0
  170 COH1=COH1+AR1(1)+.5*GIS1-SR1*50.-(.2/52.)*50.*AIBW1-(.08/52.)*TAR1
      DO 180 J=1,3
  180 AR1(J)=AR1(J+1)
      AR1(4)=.5*GIS1
      TAR1=AR1(1)+AR1(2)+AR1(3)+AR1(4)
      WNP1=WGP1-(.2/52.)*50.*AIBW1-(.08/52.)*TAR1
      IF (ADDW1-.70*FD1) 190,210,210
  190 CALL FORCST(DDP1,FD1,ADDW1)
      IF (AIEW1+PIPLIE(1)-1.5*FD1) 220,220,200
  200 PMD1=0.15
      SSF1=0.5
      GO TO 230
  210 CALL FORCST(DDP1,FD1,ADDW1)
  220 PMD1=0.0
      SSF1=0.0
  230 CONTINUE
      CALL OMDR(PIPLIE,UO1,TUS1,AIEW1,FD1,ASB1,DI1,DSB1)
      CALL PERSNL (FD,FD1,Y,Y1,MT,Z,Z1,ZT,SAL,SAL1,SUMM,SUMM1,JWKS,M,M1)
C UPDATE COH,ULS,WNP WITH PERSONNEL CONSIDERATIONS
      COH=COH-FC-SAL
      COH1=COH1-FC1-SAL1
      WNPT=(WNP+WNP1)-FC-FC1-SAL-SAL1
      JWKS2=JWKS+1
      WRITE (6,24) JWKS2,ADDW1,FD1,COH,GIS,WNPT,FD
   24 FORMAT(/I7,6X,F8.2,9X,F9.2,11X,F12.2,6X,F10.2,10X,F10.2,6X,F6.0)
      WRITE (6,35) ADDW,SAL,TUS1,TUS,WGP,ULS,ZT,Z,MT,SH
```

```
 35 FORMAT  (7X,F6.0,8X,F9.2,9X,F11.2,12X,F6.0,10X,F10.2,10X,F6.1,6X,F
   *5.2,2X,F3.1,2X,I3,2X,F5.2)
 25 CONTINUE
    STOP
    END
* FOR DEMAND
    SUBROUTINE DEMAND(ADDW,PMD,TMU,SD,SSF,V)
    D = TMU + SD * V
    IF (D) 1,2,2
  1 D = 0.0
  2 IF (PMD - 0.15) 3,4,4
  3 SSF = 0.0
    GO TO 5
  4 SSF = 0.05
  5 ADDW = D * (1.0 + SSF)
    N = ADDW
    ADDW = N
    RETURN
    END
* FOR SUPPLY
    SUBROUTINE SUPPLY (PIPLIN,SR,UO,X)
    DIMENSION PIPLIN(9)
    SR = PIPLIN(1)
    DO 2 K=1,7
  2 PIPLIN(K) = PIPLIN(K+1)
    PIPLIN(8) = 0.0
    M=X*100.
    IF (M-9) 3,3,4
  3 LT = 4
    GO TO 11
  4 IF (M-24) 5,5,6
  5 LT = 5
    GO TO 11
  6 IF (M-74) 7,7,8
  7 LT = 6
    GO TO 11
  8 IF (M-89) 9,9,10
  9 LT = 7
    GO TO 11
 10 LT = 8
 11 PIPLIN(LT) = PIPLIN(LT) + UO
    RETURN
    END
* FOR FORCAS
    SUBROUTINE FORCAS(DDP,FD,ADDW)
    DIMENSION DDP(11)
    DO 1 IB=1,9
    L = 11 - IB
  1 DDP(L) = DDP(L-1)
    DDP(1) = ADDW
    FD=(DDP(1)+DDP(2)+DDP(3))/3.0
    RETURN
    END
* FOR ORDER
    SUBROUTINE ORDER(PIPLIN,UO,TUS,AIEW,FD,ASR,DI,DSB)
    DIMENSION PIPLIN(9)
    DI = 3.0 * FD
    ASR = 0.0
    DO 1 N = 1,8
```

```
    1 ASB = ASB + PIPLIN(N)
      DSB = 6.0 * FD
      UO = IUS + .5 + (DI - AIEW + DSB - ASB)
      N=UO
      UO=N
      IF (UO) 2,3,3
    2 UO = 0.0
    3 CONTINUE
      RETURN
      END
•IT FOR RANUM,RANUM
      SUBROUTINE RANDM(DEVIOT,RND2)
      COMMON/RANO/IRD,NSTART,RND1,IRN,NOPS
      IRN=IABS(IRD**2+NOPS**2)
      IRD=(IRN-(IRN/1000000000)*1000000000)/1000
      IF(IRD.EQ.0)IRD=1
      NOPS=NOPS+1
      RND2=FLOAT(IRD)*0.000001
      DEVIOT=SQRT(-2.*ALOG(RND1))*COS(6.2831854*RND2)
      RND1=RND2
      RETURN
      END
•IT FOR INIT,INIT
      SUBROUTINE INIT
      COMMON/RANO/IRD,NSTART,RND1,IRN,NOPS
      IRD=NSTART
      RND1=FLOAT(NSTART)*0.000001
      NOPS=47436
      RETURN
      END
• FOR DEMND
      SUBROUTINE DEMND (ADDW1,PMD1,TMU1,SD1,SSF1,V)
      D = TMU1*SD1*V
      IF (D) 1,2,2
    1 D=0.0
    2 IF (PMD1-0.15) 3,4,4
    3 SSF1=0.0
      GO TO 5
    4 SSF1=0.05
    5 ADDW1=D*(1.0+SSF1)
      N=ADDW1
      ADDW1=N
      RETURN
      END
• FOR SUPPLI
      SUBROUTINE SUPPLI (PIPLIE,SR1,UO1,X)
      DIMENSION PIPLIE (9)
      SR1=PIPLIE(1)
      DO 2 K=1,7
    2 PIPLIE (K)=PIPLIE (K+1)
      PIPLIE(8)=0.0
      M=X*100.
      IF (M-9) 3,3,4
    3 LT = 4
      GO TO 11
    4 IF (M-24) 5,5,6
    5 LT=5
      GO TO 11
    6 IF (M-74) 7,7,8
```

```
    7 LT = 6
      GO TO 11
    8 IF (M-89) 9,9,10
    9 LT=7
      GO TO 11
   10 LT=8
   11 PIPLIE(LT)=PIPLIE(LT)+U01
      RETURN
      END
 * FOR FORCST
      SUBROUTINE FORCST (DDP1,FD1,ADDW1)
      DIMENSION DDP1(11)
      DO 1 IB=1,9
      L=11-IB
    1 DDP1(L)=DDP1(L-1)
      DDP1(1)=ADDW1
      FD1=(DDP1(1)+DDP1(2)+DDP1(3))/3.0
      RETURN
      END
 * FOR ORUR
      SUBROUTINE ORUR (PIPLIE,U01,TUS1,AIEW1,FD1,ASB1,DI1,DSB1)
      DIMENSION PIPLIE (9)
      DI1=3.0*FD1
      ASB1=0.0
      DO 1 N=1,8
    1 ASB1=ASB1+PIPLIE(N)
      DSB1=6.0*FD1
      U01=TUS1+.5+(DI1-AIEW1+DSB1-ASB1)
      N=U01
      U01=N
      IF (U01) 2,3,3
    2 U01=0.0
    3 CONTINUE
      RETURN
      END
 * FOR PERSNL
      SUBROUTINE PERSNL (FD,FD1,Y,Y1,MT,Z,Z1,ZT,SAL,SAL1,SUMM,SUMM1,JWKS
     *,M,M1)
      DIMENSION Y(101),Y1(101),XM(100),XM1(100),XMT(100)
      Y(JWKS+1)=FD/20.
      Y1(JWKS+1)=FD1/20.
      L2=Y(JWKS+1)+0.9999
      L1=Y(JWKS)+0.9999
      M=L2-L1
      L12=Y1(JWKS+1)+0.9999
      L11 = Y1(JWKS)+0.9999
      M1=L12-L11
      XM(JWKS+1)=M
      XM1(JWKS+1)=M1
      SUMM=SUMM+XM(JWKS)
      SUMM1=SUMM1+XM1(JWKS)
      Z=SUMM+Y(1)
      Z1=SUMM1+Y1(1)
      MT=M1+M
   45 XMT(JWKS)=MT
      ZT=Z1+Z
      IF (XM(JWKS))1,3,3
    1 SAL=Z*192.-XM(JWKS)*2.*192.
      GO TO 4
```

```
      3 SAL=Z*192.
      4 IF (XM1(JWKS)) 5,6,6
      5 SAL1=Z1*192.-XM1(JWKS)*2.*.92.
        GO TO 7
      6 SAL1=Z1*192.
      7 RETURN
        END
* XQT SIMCU
* FIN
* FIN
```

APPENDIX C

MULTIVARIATE LINEAR REGRESSION ANALYSIS

Multivariate Linear Regression Analysis[15]

The problem is how to estimate a variable or set of variables of an unknown universe given a sample set of data from that universe. The method to be used will be the least square method using a linear model. It is also assumed that the errors about the regression are normally distributed. This distribution is assumed homoscedastic; i.e., the scatter is uniform along the line of regression.

The mathematical model for the universe plane of regression of X_1 on $X_2, X_3, \ldots X_p$ is represented by the equation

$$X'_{1r} = a'_{1.23\ldots p} + b'_{12.3\ldots p} X_2 + b'_{13.2\ldots p} X_3 + \ldots + b'_{1p.23\ldots p-1} X_p \qquad (1)$$

where p is the number of independent variables. The equation for the estimate of the universe is

$$X_{1r} = a_{1.23\ldots p} + b_{12.3\ldots p} X_2 + b_{13.2\ldots p} X_3 + \ldots + b_{1p.23\ldots p-1} X_p \qquad (2)$$

with the origin at $X_1 = X_2 = X_3 = \ldots = X_p = 0$.

The primes on the variables indicate the universe variables and are estimated by the same symbol without the prime. The subscripts are divided by

a dot to indicate the dependent variable to the left and the independent variable to the right of the dot. The constant $a_{1.23\ldots p}$ shows that the dependent variable is X_1 and $X_2, X_3, \ldots X_p$ are the independent variables. In the case of $b_{12.3\ldots p}$ X_1 and X_2 are linked together for the dependent variables and $X_3 \ldots X_p$ are the independent variables.

The method of least squares requires the coefficients be determined from the condition that

$$\begin{aligned} \Sigma(X_1 - \dot{X}_{1r})^2 = \Sigma(X_1 - a_{1.23\ldots p} - \\ - b_{12.3\ldots p} X_2 - \ldots - \\ - b_{1p.23\ldots p-1} X_p)^2 \end{aligned} \tag{3}$$

be a minimum for all sample points.

This is done by taking the first derivative and setting equal to zero. Letting (3) equal $\Sigma v^2_{1.23\ldots p}$ then

$$\begin{aligned} \frac{\partial \Sigma v^2_{1.23\ldots p}}{\partial a_{1.23\ldots p}} &= 0 \\ \frac{\partial \Sigma v^2_{1.23\ldots p}}{\partial b_{12.3\ldots p}} &= 0 \\ &\ldots \\ \frac{\partial \Sigma v^2_{1.23\ldots p}}{\partial b_{1p.2\ldots p-1}} &= 0 \end{aligned} \tag{4}$$

The following equations are the result of applying the minimization

process.

$$na_{1.23\ldots p} + b_{12.3\ldots p}\Sigma X_2 + \ldots + b_{1p.23\ldots p-1}\Sigma X_p = \Sigma X_1$$

$$a_{1.23\ldots p}\Sigma X_2 + b_{12.3\ldots p}\Sigma X_2^2 + \ldots + b_{1p.23\ldots p-1}\Sigma X_2 X_p = \Sigma X_1 X_2$$

$$\ldots \qquad (5)$$

$$a_{1.23\ldots p}\Sigma X_p + b_{12.3\ldots p}\Sigma X_2 X_p + \ldots + b_{1p.23\ldots p-1}\Sigma X_p^2 = \Sigma X_1 X_p$$

These are the least-square or "normal" equations for estimating the universe regression. Another form of these equations can be obtained by shifting the origin from 0 to the mean value of the X's $(\overline{X})$.

The resulting equations are

$$b_{12.3\ldots p}\Sigma x_2^2 + \ldots + b_{1p.23\ldots p-1}\Sigma x_2 x_p = \Sigma x_1 x_2$$

$$\ldots \qquad (6)$$

$$b_{12.3\ldots p}\Sigma x_2 x_p + \ldots + b_{1p.23\ldots p-1}\Sigma x_p^2 = \Sigma x_1 x_p$$

where the origin is at $\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_p$.

The lower case x's are deviations from the mean. To solve equations (6) for the "b" coefficients it is more convenient to write the equations in matrix form and solve by use of linear algebra. The following matrices are defined as:

$$A = \begin{bmatrix} \Sigma x_2^2 & \Sigma x_2 x_3 & \cdots & \Sigma x_2 x_p \\ \Sigma x_2 x_3 & \Sigma x_3^2 & \cdots & \Sigma x_3 x_p \\ \vdots & & & \\ \Sigma x_2 x_p & \Sigma x_3 x_p & \cdots & \Sigma x_p^2 \end{bmatrix} \qquad (7)$$

$$B = \begin{bmatrix} b_{12.3\ldots p} \\ b_{13.2\ldots p} \\ \vdots \\ b_{1p.23\ldots p-1} \end{bmatrix} \qquad (8)$$

$$K = \begin{bmatrix} \Sigma x_1 x_2 \\ \Sigma x_1 x_3 \\ \vdots \\ \Sigma x_1 x_p \end{bmatrix} \qquad (9)$$

Then the matrix equation is

$$AB = K \qquad (10)$$

The solution to equation (10) is

$$B = A^{-1} K \qquad (11)$$

This analysis can be extended to more than one dependent variable.

The same procedure may be used with other dependent variables by bringing in new "K" matrices. The results are the new regression coefficients, i.e., new "B" matrices. A compact notation can result if the "K" matrix as defined in equation (10) as a column vector is augmented with more column vectors, one for each dependent variable. Then the "K" matrix has the number of rows equal to the number of independent variables and the number of columns equal to the number of dependent variables. The "B" matrix has the number of rows equal to the number of independent variables and the number of columns equal to the number of dependent variables. The "B" matrix will have the same order as the "K" matrix. The coefficients in the "B" matrix correspond column by column to the dependent variables in the "K" matrix.

The standard error of estimate can be easily determined using $\Sigma v^2_{1.23...p}$ which is equal to equation (3). The unbiased estimate of the square of the universe standard error estimate is

$$S^2_{1.23...p} = \frac{\Sigma v^2_{1.23...p}}{n - p} \qquad (12)$$

where n is the amount of data samples and p-1 is the number of independent varables.

The squared sample multiple correlation coefficient is

$$R^2_{1.23...p} = 1 - \frac{\Sigma v^2_{1.23...p}}{\Sigma x^2_1}$$

The squared sample multiple correlation coefficient may be viewed as mea-

suring the relative amount of variation in the dependent variable that is "explained" by the independent variable. The coefficient of correlation, $R_{1.23\ldots p}$, ranges from -1 to +1 where a -1 indicates a perfect negative correlation and a +1 indicates a perfect positive correlation between the dependent and independent variables. Obviously, a zero coefficient of correlation says that the variables are uncorrelated.

APPENDIX D

COMPLETE LABORATORY SYSTEM COMPUTER LISTING

DEFINITION OF VARIABLE NAMES

USED IN THE LABORATORY SYSTEM COMPUTER PROGRAM

Main Program

The main program of the laboratory system contains three major subroutines -- SIMCO, DONAFT, and DONA -- as shown in the flow diagram in Figure 4.2. There are three arrays of variables that are common block to each of the major subroutines. The parameter names are also defined.

Variable	Definition
INDEP	Number of independent variables in DONA model.
IOB	Number of weeks in SIMCO run, number of observations.
IPSI	Number of SIMCO state variables.
IXXX	Number of SIMCO inputs.
IYYY	Number of SIMCO outputs.
L	Same as INDEP.
MAX	Sum of independent and dependent variables in DONA model. (Number of independent variables = number of dependent variables in DONA model).
PSI (I,J)	An array of the SIMCO state variables where the "I" index is the time variable and the "J" index is the particular dimension of the state variables. For example, PSI (1,2) is the second state variable in the first week.
X (I,J)	An array of the SIMCO input variables with the "I" and "J" as described in PSI (I,J) above.
YY (I,J)	An array of the SIMCO output variables with

the "I" and "J" as described in PSI (I,J) above.

SIMCO Subroutine

The SIMCO subroutine and its internal subroutines are shown in the flow diagram in Figure 2.2. The internal subroutines to the SIMCO subroutine are:

DEMAND

SUPPLY

FORCAS

ORDER

RANDM

INIT

PERSNL

The following is a list of the variable names used in the SIMCO subroutine and its internal subroutines. When a "1" is used as a suffix on the variable name, the variable name applies to product two corresponding to the same variable name as for product one; eg., ADDW1 is the actual demand during the week for product two and ADDW is the actual demand during the week for product one. The letter "S" is used as a suffix to a variable name to change the original variable name to a new variable with subscripts; eg., ADDW to ADDWS (1).

Variable	Definition
A	See DEVIOT

Variable	Definition
AAIBW	Adjusted actual inventory beginning new week.
ADDW	Actual demand during the week.
AIBW	Actual inventory at the beginning of the week.
AIEW	Actual inventory at the end of the week.
AR (I)	Accounts receivable in dollars.
ASB	Actual supply backorder.
B	See RND2.
BS	Backorder sales at the beginning of the week.
CBQ	Customer backorder queue.
COH	Cash on hand.
CS	Cash sales.
DDP (I)	Demand during the past I week .
DEVIOT	Standard normal distributed number (mean = 0 standard deviation = 1).
DI	Desired inventory.
DSB	Desired supply backorder.
FC	Fixed costs in dollars.
FD	Forecasted demand.
GIS	Gross income from the week's sales.
M	Change in number of salesmen for coming week.
MT	Total change in number of salesmen, product 1 and 2.

Variable	Definition
NSTART	Any 6 digit number used in the random number generator.
PIPLIE (K)	Quantity in pipeline to be delivered in K week, product 2.
PIPLIN (K)	Quantity in pipeline to be delivered in K week, product 1.
PMD	Price markdown.
RND2	Uniformly distributed random number between 0 and 1.
SAL	Salaries for personnel.
SD	Standard deviation of demand.
SR	Stock received.
SSF	Sales stimulation factors.
SUMM	Change in number of personnel in the current week.
TAR (I)	Total accounts receivable.
TMU	The mean weekly demand for units.
TUS	Total units sold during the week.
ULS	Units of lost sales.
UO	Quantity of units ordered.
V	See DEVIOT.
WC	Wholesale cost.
WGP	Weekly gross profit.
WNP	Weekly net profit.

Variable	Definition
WNPT	Weekly net profit total of product 1 and 2.
X	See RND2.
XM	See M.
Y	Number of salesmen for coming week.
Z	Number of salesmen for current week product 1.
ZT	Total number of salesmen for current week product 1 and 2.

DONAFT Subroutine

The DONAFT subroutine and its internal subroutines are shown in the flow diagram in Figure 4.3. The internal subroutines to the DONAFT subroutine are:

MULTR

MEANS

SUMSQ

INV3 and ADJUST

MXMLT

The variable names used in these subroutines are defined as follows:

Variable	Definition
AAA	A working matrix for the matrix transformation into intercorrelations.
AUK	An array that contains the sum of squares of the deviations.
B	The matrix of weights.

Variable	Definition
BO	Weight of the intercept.
C	The upper left quarter of the CUB matrix.
CUB	The inverse sum of squares matrix.
DELPSI	Backward difference of the SIMCO state variables.
DELX	Backward difference of the SIMCO input variables.
DELY	Backward difference of the SIMCO output variables.
DET	Determinant of intercorrelation matrix.
DTAPSI	Forward difference of the SIMCO state variables.
MR	The squared coefficient of multiple correlation.
RSQ	Coefficient of multiple correlation.
S	"S" matrix of the DONA model.
SB	The weight standard deviation matrix.
SBO	Standard deviation of the intercept.
SEE	Standard error of the estimate array
T	Student t statistic.
UB	The upper right quarter of the CUB matrix.
UK	The upper right quarter of the AUK matrix.
XBAR	The means of X data matrix.
XX (I, J)	An array of the forward and backward difference of SIMCO variables, also X data matrix used as an input to MULTR.

DONA Subroutine

The DONA subroutine is shown in the flow diagram in Figure 4.4. The variable names used in this subroutine are defined as follows:

Variable	Definition
R (I,1)	Vector of independent variables in the DONA model. The "I" index denotes the particular variable.
RR (I,1,J)	Same as R (I,1) except the "J" index is the time variable.
S	"S" matrix of the DONA model.
T (I 1)	Vector of dependent variables in the DONA model. The "I" index denotes the particular variable.
TT (I,1,J)	Same as T (I,1) except the "J" index is the time variable.

```
* FOR MAIN
C  PROGRAM FOR SIMCO AND DONA LABORATORY SYSTEM
      CALL SIMCO
      CALL DONAFT
      CALL DONA
      STOP
      END
* FOR SIMCO
      SUBROUTINE SIMCO
C     PROGRAM SIMCO
      PARAMETER IXXX=2,IYYY=10,IPSI=30,IOB=82,INDEP=7,MAX=14
C IXXX=NO. OF INPUTS, IYYY=NO.OF OUTPUTS, IPSI=NO. OF STATE VARIABLES,
C IOB = NO. OF OBSERVATIONS,INDEP = NO. OF INDEPENDENT VARIABLES,MAX =
C TWO TIMES NO. OF INDEPENDENT VARIABLES (INDEP=DEP).
      COMMON/BLK2/PSI(IOB,IPSI),X(IOB,IXXX),YY(IOB,IYYY)
      DIMENSION AR(5),PIPLIN(9),DOP(11),Y(IOB),XM(IOB),AR1(5),PIPLIE(9),
     1DOP1(11),Y1(IOB),XM1(IOB)
      DIMENSION UOS(IOB),UOS1(IOB),ULSS(IOB),ULSS1(IOB),AIEWS(IOB),AIEWS
     *1(IOB),CHOS(IOB),CHOS1(IOB),FDS(IOB),FDS1(IOB),TARS(IOB),TARS1(IOB
     *),COHS(IOB),COHS1(IOB),ZS(IOB),ZS1(IOB),MTS(IOB),SRS(IOB),SRS1(IOB
     *),AIBWS(IOB),AIBWS1(IOB),BSS(IOB),BSS1(IOB),AAIBWS(IOB),AUIBWS(IOB
     *),ADDWS(IOB),ADDWS1(IOB),TUSS(IOB),TUSS1(IOB),GISS(IOB),GISS1(IOB)
     *,WGPS(IOB),WGPS1(IOB),FCS(IOB),FCS1(IOB),WNPS(IOB),WNPS1(IOB),ASBS
     *(IOB),ASBS1(IOB),DIS(IOB),DIS1(IOB),DSBS(IOB),DSBS1(IOB),CSS(IOB),
     *CSS1(IOB),WCS(IOB),WCS1(IOB),SALS(IOB),SALS1(IOB),XIVS(IOB),XIVS1(
     *IOB),ZTS(IOB),MS(IOB),MS1(IOB),WNPTS(IOB)
      EQUIVALENCE (UOS(1),PSI(1,1)),(UOS1(1),PSI(1,2)),(ULSS(1),PSI(1,3)
     1),(ULSS1(1),PSI(1,4)),(AIEWS(1),PSI(1,5)),(AIEWS1(1),PSI(1,6)),
     2(CHOS(1),PSI(1,7)),(CHOS1(1),PSI(1,8)),(FDS(1),PSI(1,9)),(FDS1(1),
     3PSI(1,10)),(COHS(1),PSI(1,11)),(COHS1(1),PSI(1,12)),(SRS(1),PSI
     4(1,13)),(SRS1(1),PSI(1,14)),(AIBWS(1),PSI(1,15)),(AIBWS1(1),PSI(1,
     516)),(BSS(1),PSI(1,17)),(BSS1(1),PSI(1,18)),(TUSS(1),PSI(1,19)),(T
     6USS1(1),PSI(1,20)),(WGPS(1),PSI(1,21)),(WGPS1(1),PSI(1,22)),(WNPS(
     71),PSI(1,23)),(WNPS1(1),PSI(1,24)),(DIS(1),PSI(1,25)),(DIS1(1),PSI
     8(1,26)),(ZS(1),PSI(1,27)),(ZS1(1),PSI(1,28)),(MS(1),PSI(1,29)),(MS
     91(1),PSI(1,30))
      EQUIVALENCE (SALS(1),YY(1,1)),(WCS(1),YY(1,2)),(CSS(1),YY(1,3)),
     1(TARS(1),YY(1,4)),(FCS(1),YY(1,5)),(SALS1(1),YY(1,6)),(WCS1(1),YY
     2(1,7)),(CSS1(1),YY(1,8)),(TARS1(1),YY(1,9)),(FCS1(1),YY(1,10))
      EQUIVALENCE (ADDWS(1),X(1,1)),(ADDWS1(1),X(1,2))
      COMMON/RAND/IRD,NSTART,RND1,IRN,NOPS
      NSTART=123456
      CALL INIT
    3 FORMAT (///5X,4HWEEK,4HADDW,4X,5HADDW1,6X,3HSAL,6X,2HWC,8X,4HTUS1,
     *   6X,3HCOH,5X,3HTUS,6X,3HGIS,7X,3HWGP,7X,3HWNP,4X,3HULS,5X,2HFD,3X
     *,2HZ1,3X,1HZ,3X,2HMT,4X,3H SR/)
C  PRINT COLUMN HEADINGS
      WRITE (6,3)
C INITIALIZE VALUES
C INITIALIZE DATA ACCUMULATION VALUES AT ZERO
C INITIALIZE PIPELINE FOR THE NEXT 8 WEEKS
      DO 5 M=1,6
    5  PIPLIN(M)=50.0
      DO 50 M=1,6
   50 PIPLIE(M)=50.0
      PIPLIN(7)=0.0
      PIPLIE(7)=0.0
      PIPLIN(8)=0.0
```

```
      PIPLIE(8)=0.0
      TMU = 50.0
      TMU1=50.0
      SU=20.0
      SU1=15.0
      UO=50.0
      UO1=50.0
      ULS = 0.0
      ULS1=0.0
      AIEW=150.0
      AIEW1=150.0
      CBO = 0.0
      CBO1=0.0
      DO 6 J = 1,10
    6 UDP(J)=50.0
      DO 60 J=1,10
   60 UDP1(J)=50.0
      FC=50.0
      FC1=50.0
      PRO = 0.0
      PRO1=0.0
      SSF = 0.0
      SSF1=0.0
C INITIALIZE ACCOUNTS RECEIVABLE FOR THE NEXT 4 WEEKS
      DO 7 K=1,4
    7 AR(K)=2500.0
      DO 70 K=1,4
   70 AR1(K)=2500.0
      TAR=10000.0
      TAR1=10000.0
      COH=4000.0
      COH1=4000.0
C INITIALIZE PERSONNEL ACTIONS
      SUMW = 0.0
      SUMW1=0.0
      Y(1)=3.0
      Y1(1)=3.0
      ULSA=0.0
      ULSA1=0.0
      XM(1)=0.0
      XM1(1)=0.0
C RUN FOR 100 WEEKS
      IH=100-1
      DO 25 JWKS=1,IH
      CALL RANDM(A,B)
C CALL SUPPLY TO DETERMINE LEAD TIME FOR UNITS ORDERED
      CALL SUPPLY(PIPLIN,SR,UO,B)
      WC=SR*50.
C FILL CUSTOMER BACKORDERS IF ANY
      AIBW = AIEW + SR
      IF (CBO - AIBW) 8,8,9
    8 BS = CBO
      AAIBW = AIBW - CBO
      CBO = 0.0
      ULS = 0.0
      GO TO 10
    9 BS = AIBW
      AAIBW = 0.0
      ULS = (CBO - AIBW) * .20
```

```
      CBQ = (CBQ - AIBW) * .80
   10 N = CBQ
      CBQ = N
      N = ULS
      ULS = N
C CALL DEMAND TO DETERMINE DEMAND FOR THIS WEEK
      CALL DEMAND(ADDW,PMD,TMU,SD,SSF,A)
C DETERMINE TOTAL SALES DURING THE WEEK
      IF (ADDW - AAIBW) 11,11,12
   11 TUS = BS + ADDW
      GO TO 13
   12 TUS = BS + AAIBW
C DETERMINE BACKORDER QUEUE LENGTH
      CBQ = CBQ + (ADDW - AAIBW)
C DETERMINE GROSS INCOME,GROSS PROFIT, AND ENDING INVENTORY
   13 GIS = 100. * (1.0 - PMD) * TUS
      CS=0.5*GIS
      WGP = GIS - 50.0 * TUS
      AIEW = AAIBW - ADDW
      IF (AIEW) 14,15,15
   14 AIEW = 0.0
   15 CONTINUE
      XIV=AIEW*50.
   16 FC=777.0
C DETERMINE CASH ON HAND
   17 COH=COH+AR(1)+.5*GIS-SR*50.-(.2/52.)*50.*AIBW-(.08/52.)*TAR
C UPDATE ACCOUNTS RECEIVABLE AND DETERMINE TOTAL A.R.
      DO 18 J = 1,3
   18 AR(J) = AR(J+1)
      AR(4) = .5 * GIS
      TAR = AR(1) + AR(2) + AR(3) + AR(4)
C DETERMINE WEEKLY NET PROFIT
      WNP = WGP - (.2/52.)*50. * AIBW - (.08/52.) * TAR
      IF (ADDW -.70 * FD) 19,21,21
C FORECAST DEMAND FOR THE COMING WEEK AND DETERMINE IF A PRICE MARKDOWN/
C IS NECESSARY
   19 CALL FORCAS(DDP,FD,ADDW)
      IF (AIEW + PIPLIN(1) - 1.5 * FD) 22,22,20
   20 PMD=0.0
      SSF=0.0
      GO TO 23
   21 CALL FORCAS (DDP,FD,ADDW)
   22 PMD = 0.0
      SSF = 0.0
   23 CONTINUE
C CALL ORDER TO DETERMINE THE NUMBER OF UNITS TO BE ORDERED
      CALL ORDER (PIPLIN,UO,TUS,AIEW,FD,ASB,DI,DSB)
      CALL RANDM(A,B)
      CALL SUPPLY(PIPLIE,SR1,UO1,B)
      WC1=SR1*50.
      AIBW1=AIEW1+SR1
      IF(CBQ1-AIBW1) 80,80,90
   80 BS1=CBQ1
      AAIBW1=AIBW1-CBQ1
      CBQ1=0.0
      ULS1=0.0
      GO TO 100
   90 BS1=AIBW1
      AAIBW1=0.0
```

```
      ULS1=(CBQ1-AIBW1)*.20
      CBQ1=(CBQ1-AIBW1)*.80
  100 N=CBQ1
      CBQ1=N
      N=ULS1
      ULS1=N
      CALL DEMAND (ADDW1,PMD1,TMU1,SD1,SSF1,A)
      IF (ADDW1-AAIBW1) 110,110,120
  110 TUS1=BS1+ADDW1
      GO TO 130
  120 TUS1=BS1+AAIBW1
      CBQ1=CBQ1+(ADDW1-AAIBW1)
  130 GIS1=100.*(1.0-PMD1)*TUS1
      CS1=0.5*GIS1
      WGP1=GIS1-50.0*TUS1
      AIEW1=AAIBW1-ADDW1
      IF (AIEW1) 140,150,150
  140 AIEW1=0.0
  150 CONTINUE
      XIV1=AIEW1*50.
  160 FC1=777.0
  170 COH1=COH1+AR1(1)+.5*GIS1-SR1*50.-(.2/52.)*50.*AIBW1-(.08/52.)*TAR1
      DO 180 J=1,3
  180 AR1(J)=AR1(J+1)
      AR1(4)=.5*GIS1
      TAR1=AR1(1)+AR1(2)+AR1(3)+AR1(4)
      WNP1=WGP1-(.2/52.)*50.*AIBW1-(.08/52.)*TAR1
      IF (ADDW1-.70*FD1) 190,210,210
  190 CALL FORCAS(UUP1,FD1,ADDW1)
      IF (AIEW1+PIPLIE(1)-1.5*FD1) 220,220,200
  200 PMD1=0.0
      SSF1=0.0
      GO TO 230
  210 CALL FORCAS(UUP1,FD1,ADDW1)
  220 PMD1=0.0
      SSF1=0.0
  230 CONTINUE
      CALL PERSNL (FD,FD1,Y,Y1,MT,Z,Z1,ZT,SAL,SAL1,SUMM,SUMM1,JWKS,M,M1)
C UPDATE COH,ULS,WNP WITH PERSONNEL CONDISERATIONS
      COH=COH-FC-SAL
      COH1=COH1-FC1-SAL1
      WNPT=(WNP+WNP1)-FC-FC1-SAL-SAL1
      CALL ORDER(PIPLIE,UO1,TUS1,AIEW1,FD1,ASB1,OI1,OSB1)
      JWKS2=JWKS+1
C  PRINT RESULTS
      WRITE (6,24) JWKS2,ADDW1,WC,COH,GIS,WNPT,FO
   24 FORMAT(/I7,6X,F8.2,9X,F9.2,11X,F12.2,6X,F10.2,10X,F10.2,6X,F6.0)
      WRITE (6,35) ADDW,SAL,TUS1,TUS,WGP,ULS,ZT,Z,MT,SR
   35 FORMAT  (7X,F6.0,8X,F9.2,9X,F11.2,12X,F6.0,10X,F10.2,10X,F6.1,6X,F
     *5.2,2X,F3.1,2X,I3,2X,F5.2)
      WNPTS(JWKS)=WNPT
      UOS1(1)=50.0
      UOS(1)=50.0
      SALS(1)=500.0
      WCS(1)=3000.0
      TUSS(1)=50.0
      ADDWS(1)=50.0
      ADDWS1(1)=40.0
      ULSS(1)=0.0
```

```
ULSS1(1)=0.0
AIEWS(1)=150.0
AIEWS1(1)=150.0
CBOS(1)=0.0
CBOS1(1)=0.0
FOS(1)=50.0
FOS1(1)=50.0
TARS(1)=10000.0
TARS1(1)=10000.0
COMS(1)=4000.0
COMS1(1)=4000.0
ZS(1)=0.0
ZS1(1)=0.0
MTS(1)=0.0
SHS(1)=0.0
SHS1(1)=0.0
AIBWS(1)=0.0
AIBWS1(1)=0.0
BSS(1)=0.0
BSS1(1)=0.0
AAIBWS(1)=0.0
AUIBWS(1)=0.0
TUSS1(1)=0.0
GISS(1)=0.0
GISS1(1)=0.0
WGPS(1)=0.0
WGPS1(1)=0.0
FCS(1)=0.0
FCS1(1)=0.0
WRPS(1)=0.0
WRPS1(1)=0.0
ASBS(1)=0.0
ASBS1(1)=0.0
DIS(1)=0.0
DIS1(1)=0.0
DSBS(1)=0.0
DSBS1(1)=0.0
CSS(1)=0.0
CSS1(1)=0.0
WCS1(1)=0.0
SALS1(1)=0.0
XIVS(1)=0.0
XIVS1(1)=0.0
ZTS(1)=0.0
MS(1)=0.0
MS1(1)=0.0
UOS(JWKS+1)=UO
UOS1(JWKS+1)=UO1
ULSS(JWKS+1)=ULS
ULSS1(JWKS+1)=ULS1
AIEWS(JWKS+1)=AIEW
AIEWS1(JWKS+1)=AIEW1
CBOS(JWKS+1)=CBO
CBOS1(JWKS+1)=CBO1
FOS(JWKS+1)=FO
FOS1(JWKS+1)=FO1
TARS(JWKS+1)=TAR
TARS1(JWKS+1)=TAR1
COMS(JWKS+1)=COM
```

```
      COMS1(JWKS+1)=COM1
      ZS(JWKS+1)=Z
      ZS1(JWKS+1)=Z1
      MTS(JWKS+1)=MT
      SRS(JWKS+1)=SR
      SRS1(JWKS+1)=SR1
      AIBWS(JWKS+1)=AIBW
      AIBWS1(JWKS+1)=AIBW1
      BSS(JWKS+1)=BS
      BSS1(JWKS+1)=BS1
      AAIBWS(JWKS+1)=AAIBW
      AUIBWS(JWKS+1)=AAIBW1
      ADDWS(JWKS+1)=ADDW
      ADDWS1(JWKS+1)=ADDW1
      TUSS(JWKS+1)=TUS
      TUSS1(JWKS+1)=TUS1
      GISS(JWKS+1)=GIS
      GISS1(JWKS+1)=GIS1
      WGPS(JWKS+1)=WGP
      WGPS1(JWKS+1.=WGP1
      FCS(JWKS+1)=FC
      FCS1(JWKS+1)=FC1
      WNPS(JWKS+1)=WNP
      WNPS1(JWKS+1)=WNP1
      ASBS(JWKS+1)=ASB
      ASBS1(JWKS+1)=ASB1
      OIS(JWKS+1)=OI
      OIS1(JWKS+1)=OI1
      OSBS(JWKS+1)=OSB
      OSBS1(JWKS+1)=OSB1
      CSS(JWKS+1)=CS
      CSS1(JWKS+1)=CS1
      WCS(JWKS+1)=WC
      WCS1(JWKS+1)=WC1
      SALS(JWKS+1)=SAL
      SALS1(JWKS+1)=SAL1
      XIVS(JWKS+1)=XIV
      XIVS1(JWKS+1)=XIV1
      ZTS(JWKS+1)=ZT
      MS(JWKS+1)=M
      MS1(JWKS+1)=M1
   25 CONTINUE
      RETURN
      SUBROUTINE DEMAND(ADDW,PMD,TMU,SU,SSF,V)
      D = TMU + SU + V
      IF (D) 1,2,2
    1 D = 0.0
    2 IF (PMD - 0.15) 3,4,4
    3 SSF = 0.0
      GO TO 5
    4 SSF = 0.05
    5 ADDW = D * (1.0 + SSF)
      N = ADDW
      ADDW = N
      RETURN
      SUBROUTINE SUPPLY (PIPLIN,SR,UO,X)
      DIMENSION PIPLIN(9)
C     DETERMINE SHIPMENTS RECEIVED
      SR = PIPLIN(1)
```

```
C   UPDATE SUPPLY
      DO 2 K=1,7
    2 PIPLIN(K) = PIPLIN(K+1)
      PIPLIN(8) = 0.0
C   DETERMINE LEAD TIME
      M=X*100.
      IF (M-9) 3,3,4
    3 LT = 4
      GO TO 11
    4 IF (M-24) 5,5,6
    5 LT = 5
      GO TO 11
    6 IF (M-74) 7,7,8
    7 LT = 6
      GO TO 11
    8 IF (M-89) 9,9,10
    9 LT = 7
      GO TO 11
   10 LT = 8
C   INCREASE APPROPRIATE WEEKS SUPPLY ORDERED BY UNITS ORDERED THIS WEEK
   11 PIPLIN(LT) = PIPLIN(LT) + UO
      RETURN
      SUBROUTINE FORCAS(DDP,FD,ADDW)
      DIMENSION DDP(11)
C   CONNLATE DEMAND FOR THE LAST TEN WEEKS
      DO 1 IB=1,9
      L = 11 - IB
    1 DDP(L) = DDP(L-1)
      DDP(1) = ADDW
C   CALCULATE FORECASTED DEMAND
      FD=(DDP(1)+DDP(2)+DDP(3))/3.0
      RETURN
      SUBROUTINE ORDER(PIPLIN,UO,TUS,AIEW,FD,ASB,DI,DSB)
      DIMENSION PIPLIN(9)
C   CALCULATE DESIRED INVENTORY
      DI = 3.0 * FD
C CALCULATE SUPPLY BACKORDER
      ASB = 0.0
      DO 1 N = 1,8
    1 ASB = ASB + PIPLIN(N)
C CALCULATE DESIRED SUPPLY BACKORDER
      DSB = 6.0 * FD
      UO = TUS + .5 + (DI - AIEW + DSB - ASB)
      N=UO
      UO=N
      IF (UO) 2,3,3
    2 UO = 0.0
    3 CONTINUE
      RETURN
      SUBROUTINE RANDOM(DEVIOT,RND2)
      COMMON/RAND/IRD,NSTART,RND1,IRN,NOPS
      IRN=IABS(IRD**2+NOPS**2)
      IRD=(IRN-(IRN/1000000000)*1000000000)/1000
      IF(IRD.EQ.0)IRD=1
      NOPS=NOPS+1
      RND2=FLOAT(IRD)*0.000001
      DEVIOT=SQRT(-2.*ALOG(RND1))*COS(6.2831854*RND2)
      RND1=RND2
      RETURN
```

```
      SUBROUTINE INIT
      COMMON/RANO/IRO,NSTART,RND1,IRN,NOPS
      IRO=NSTART
      RND1=FLOAT(NSTART)*0.000001
      NOPS=47436
      RETURN
      SUBROUTINE PERSNL (FO,FO1,Y,Y1,MT,Z,Z1,ZT,SAL,SAL1,SUMM,SUMM1,JWKS
     *,M,M1)
      DIMENSION Y(101),Y1(101),XM(100),XM1(100),XMT(100)
      Y(JWKS+1)=FO/20.
      Y1(JWKS+1)=FO1/20.
      L2=Y(JWKS+1)+0.9999
      L1=Y(JWKS)+0.9999
      M=L2-L1
      L12=Y1(JWKS+1)+0.9999
      L11 = Y1(JWKS)+0.9999
      M1=L12-L11
      XM(JWKS+1)=M
      XM1(JWKS+1)=M1
      SUMM=SUMM+XM(JWKS)
      SUMM1=SUMM1+XM1(JWKS)
      Z=SUMM+Y(1)
      Z1=SUMM1+Y1(1)
      MT=M1+M
   45 XMT(JWKS)=MT
      ZT=Z1+Z
      IF (XM(JWKS)) 1,3,3
    1 SAL=Z*192.-XM(JWKS)*2.*192.
      GO TO 4
    3 SAL=Z*192.
    4 IF (XM1(JWKS)) 5,6,6
    5 SAL1=Z1*192.-XM1(JWKS)*2.*192.
      GO TO 7
    6 SAL1=Z1*192.
    7 RETURN
      END
 .FOR DONAFT
      SUBROUTINE DONAFT
      PARAMETER IXXX=2,IYYY=10,IPSI=30,IOB=82,INDEP=7,MAX=14
      COMMON/BLK1/COB(MAX,MAX)
      COMMON/BLK2/PSI(IOB,IPSI),X(IOB,IXXX),YY(IOB,IYYY)
      COMMON/BLK3/S(INDEP,INDEP)
      DIMENSION DELPSI(IOB,IPSI),DIAPSI(IOB,IPSI),DELX(IOB,IYYY),DELY(IO
     1B,IYYY)
      DIMENSION XX(MAX,IOB)
C---------------------------------------------------------------------
C THE FOLLOWING CALCULATES DEL PSI,DELTA PSI,DEL X,DEL Y
      IB=IOB-1
      DO 1000 I=1,IPSI
      DO 1010 J=2,IB
      DELPSI(J,I)=PSI(J,I)-PSI(J-1,I)
      DIAPSI(J,I)=PSI(J+1,I)-PSI(J,I)
 1010 CONTINUE
 1000 CONTINUE
      DO 1002 I=1,IXXX
      DO 1012 J=2,IB
      DELX(J,I)=X(J,I)-X(J-1,I)
 1012 CONTINUE
 1002 CONTINUE
```

```
      DO 1003 I=1,IYYY
      DO 1013 J=2,18
      DELY(J,I)=YY(J,I)-YY(J-1,I)
 1013 CONTINUE
 1003 CONTINUE
      WRITE (6,2000)
 2000 FORMAT (1H1,60X,'TIME'/13X,1H2,8X,1H3,8X,1H4,8X,1H5,9X,1H6,8X,1H7,
     18X,1H8,8X,1H9,7X,2H10,7X,2H11,7X,2H12,7X,2H13,//,35X,'----- DEL PSI
     2, BACKWARD DIFFERENCE OF STATE VARIABLES -----')
      WRITE (6,2001)(DELPSI(J,19),J=2,13)
 2001 FORMAT (' TOS',3X,12F9.2)
      WRITE (6,2002)(DELPSI(J,9),J=2,13)
 2002 FORMAT (' FD',4X,12F9.2)
      WRITE (6,2003)
 2003 FORMAT (///41X,'----- DEL X, BACKWARD DIFFERENCE OF INPUTS -----')
      WRITE (6,2004)((DELX(J,K),J=2,13),K=1,2)
 2004 FORMAT (' ALDWS',1X,12F9.2,/,' ADDWSI',12F9.2)
      WRITE (6,2005)
 2005 FORMAT (///35X,'----- DELTA PSI, FORWARD DIFFERENCE OF STATE VARIA
     1BLES -----')
      WRITE (6,2001)(DTAPSI(J,19),J=2,13)
      WRITE (6,2002)(DTAPSI(J,9),J=2,13)
      WRITE (6,2006)
 2006 FORMAT (///41X,'----- DEL Y, BACKWARD DIFFERENCE OF OUTPUTS -----'
     1)
      WRITE (6,2007)((DELY(J,K),J=2,13),K=1,2)
 2007 FORMAT (' SAL',3X,12F9.2,/,' WC',12F10.2)
C-----------------------------------------------------------------------
C THE FOLLOWING FORMS THE X DATA MATRIX FROM SIMCO DELS AND DELTAS.
      NN=IOB-2
      DO 1300 J=1,NN
      XX(1,J)=DELPSI(J+1,1)
      XX(2,J)=DELPSI(J+1,2)
      XX(3,J)=DELPSI(J+1,5)
      XX(4,J)=DELPSI(J+1,9)
      XX(5,J)=DELPSI(J+1,10)
      XX(6,J)=DELX(J+1,1)
      XX( 7,J)=DELX(J+1,2)
      XX( 8,J)=DTAPSI(J+1,1)
      XX( 9,J)=DTAPSI(J+1,2)
      XX(10,J)=DTAPSI(J+1,5)
      XX(11,J)=DTAPSI(J+1,9)
      XX(12,J)=DTAPSI(J+1,10)
      XX(13,J)=DELY(J+1,1)
      XX(14,J)=DELY(J+1,2)
 1300 CONTINUE
      WRITE (6,801)
  801 FORMAT (1H1,55X,' X MATRIX')
      IU=MAX
      JU=NN
      KU=((IU-1)/8)+1
      DO 101 K=1,KU
      KL=8*K-7
      KP=8*K
      WRITE (6,100)((XX(I,J),I=KL,KP),J=1,JU)
  100 FORMAT (1X,8G16.7)
      WRITE (6,101)
  101 FORMAT (1H1)
      CALL MULTR (XX,INDEP,INDEP,NN,MAX)
```

```
C   THE FOLLOWING TAKES THE RIGHT HALF OF THE ''CUB'' MATRIX AND TRANPOSES
C   IT, GIVING THE ''S'' MATRIX.  THE ''S'' MATRIX IS SIZE INDEP X INDEP
      DO 1313 I=1,INDEP
      INT=INDEP+1
      DO 1313 J=INT,MAX
 1313 S(J-INDEP,I)=CUB(I,J)
      WRITE (6,4444)
 4444 FORMAT (1H1,55X,' S MATRIX')
      WRITE (6,4445) ((S(I,J),J=1,INDEP),I=1,INDEP)
 4445 FORMAT (1H ,7G13.5)
      RETURN
      END
@ FOR MULTR
      SUBROUTINE MULTR (X,KKK,M,N,MAX)
      PARAMETER ICOLS=80,IROWS=14,L=7
      REAL XR
C     (IROWS) MUST NOT BE GREATER THAN (ICOLS)                         MR020
      DIMENSION C(L,L),UK(L,L),UB(L,L)
      COMMON          A(L,L),AR(IROWS,IROWS),ARSQ(IROWS,IROWS),XBAR(IROW
     1S),AUK(IROWS,IROWS),SEE(IROWS),SBO(IROWS),                   BU(I
     2ROWS),TU(IROWS),MR(IROWS)
      DIMENSION SH(IROWS,IROWS),T(IROWS,IROWS),AAA(IROWS,IROWS)
      DIMENSION X(IROWS,ICOLS),B(IROWS,IROWS)
      COMMON/BLK 1/CUB(IROWS,IROWS)
      EQUIVALENCE (A,B),(AR,SB),(ARSQ,T)
C     (L) = NUMBER INDEPENDENT VARIABLES, (M) = NUMBER DEPENDENT        MR070
C     VARIABLES, (N) = NUMBER OBSERVATIONS                             MR070
      IF (L.LT.N) GO TO 110                                            MR090
      WRITE (6,120)                                                    MR100
  120 FORMAT(T5,'ERROR -- NOT ENOUGH DATA')                            MR110
  999 STOP                                                             MR550
  110 CONTINUE                                                         MR130
C     DATA ITEMS READ IN ONE OBSERVATION OF ALL VARIABLES AT A         MR150
C     TIME.  (X) IS THE DATA MATRIX.                                   MR150
      CALL MEANS (X, XBAR, MAX, N,IROWS)                               MR170
      CALL SUMSQ(X,AUK,MAX,N,IROWS)                                    MR180
C     (XBAR) CONTAINS THE MEANS OF (MAX) VARIABLES  (AUK)CONTAINS      MR190
C     THE SUMS OF SQUARES OF THE DEVIATIONS, (X) CONTAINS THE          MR190
C     DEVIATIONS.  (SEE) IS THE STANDARD ERROR OF ESTIMATE, (CUB) IS THE  MR190
C     INVERSE SS MATRIX, (B) IS THE MATRIX OF WEIGHTS, (SB IS THE WEIGHT  MR190
C     STD DEV MATRIX                                                   MR190
      WRITE (6,915)
  915 FORMAT (1H1,60X,' AUK MATRIX')
      JU1=MAX
      IU1=MAX
      KU1=((JU1-1)/8)+1
      DO 913 K=1,KU1
      KL1=8*K-7
      KP1=8*K
      WRITE (6,100)((AUK(I,J),J=KL1,KP1),I=1,IU1)
      WRITE (6,101)
  913 CONTINUE
      DO 413 I=1,MAX
      DO 413 J=1,MAX
  413 AAA(I,J)=AUK(I,J)
C THE FOLLOWING REPLACES ZEROES WITH EPSILON ON THE MAIN DIAGONAL OF AUK
      IF (AAA(I,I))813,814,814
  813 IF (AAA(I,I).GT.-1.E-6) AAA(I,I)=-1.E-6
      GO TO 333
```

```
814 IF (AAA(I,I).LT.1.E-6) AAA(I,I)=1.E-6
333 CONTINUE
    WRITE (6,916)
916 FORMAT (60X,' AAA MATRIX')
    DO 914 K=1,KU1
    KL2=8*K-7
    KP2=8*K
    WRITE (6,100)((AAA(I,J),J=KL2,KP2),I=1,IU1)
    WRITE (6,101)
914 CONTINUE
C   THE FOLLOWING FORMS THE INTERCORRELATION COEFFICIENTS IN MATRIX COB
C     DO 150 I=1,MAX
      DO 150 J=1,I
      COB(I,J)=AAA(I,J)/(SQRT(AAA(I,I)*AAA(J,J)))
150 COB(J,I)=COB(I,J)
C IF ABS OF ANY NUMBER IS LESS THAN -1.E-6,SET IT EQUAL TO 1.E-6.
    DO 810 J=1,MAX
    DO 810 I=1,MAX
    IF (COB(I,J)) 811,810,812
811 IF (COB(I,J).GT.-1.E-6) COB(I,J)=-1.E-6
    GO TO 810
812 IF (COB(I,J).LT.1.E-6) COB(I,J)=1.E-6
810 CONTINUE
    WRITE (6,801)
801 FORMAT (1H1,28X,'INTERCORRELATION MATRIX')
    JU=MAX
    IU=MAX
    KU=((JU-1)/8)+1
    DO 101 K=1,KU
    KL=8*K-7
    KP=8*K
    WRITE (6,100)((COB(I,J),J=KL,KP),I=1,IU)
100 FORMAT (1X,8G16.7)
    WRITE (6,101)
101 FORMAT (1H1)
    DO 820 I=1,L
    DO 820 J=1,L
820 A(I,J)=COB(I,J)
    CALL INVS (A,C,L,DET)
    WRITE(6,8870)DET
8870 FORMAT (1H1,'DETERMINANT OF A =',F20.8)
    WRITE (6,8867)
8867 FORMAT (1H0,19X,' IMPROVED INVERSE MATRIX A (INTERCORRELATED FORM)
   *')
    WRITE (6,8868)((C(I,J),J=1,L),I=1,L)
8868 FORMAT (1H ,7G13.5)
    DO 8888 I=1,L
    DO 8888 J=1,L
8888 C(I,J)=C(I,J)/(SQRT(AAA(I,I)*AAA(J,J)))
    WRITE (6,6802)
6802 FORMAT (1H1,23X,' IMPROVED INVERSE MATRIX A (NO INTERCORRELATIONS)
   *')
    WRITE (6,6803)((C(I,J),J=1,7),I=1,7)
6803 FORMAT (1H ,7G13.5)
    DO 10001 I=1,L
    DO 10001 J=1,L
10001 UK(I,J)=AUK(I,J+L)
    WRITE (6,6810)
6810 FORMAT (1H1,20X,' K MATRIX')
```

```
      WRITE (6,6803) ((UK(I,J),J=1,L),I=1,L)
      CALL MXMLT (C,UK,UB,L,L,L,L,L)
      WRITE (6,6815)
 6815 FORMAT (1H1,28X,' UB MATRIX')
      WRITE (6,6803) ((UB(I,J),J=1,L),I=1,L)
C     RECONSTRUCT THE CUB MATRIX
      DO 829 I=1,L
      DO 829 J=1,L
  829 CUB(I,J)=C(I,J)
      DO 830 I=1,L
      DO 830 J=1,L
  830 CUB(I,J+L)=UB(I,J)
  181 NSW=0
      IF(N.EQ.L+1) NSW=1                                                MR340
C     CALCULATING STANDARD DEVIATIONS AND INTERCEPTS                    MR360
      INIT=L+1                                                          MR369
      DO 220 J=INIT,MAX                                                 MR370
      K=J-L                                                             MR375
      BO(K) = XBAR(J)                                                   MR378
      SEE(K)=AUK(J,J)                                                   MR380
      DO 230 I=1,L                                                      MR390
      BO(K)=BO(K)-CUB(I,J)*XBAR(I)                                      MR395
  230 SEE(K)=SEE(K)-CUB(I,J)*AUK(I,J)                                   MR400
      MR(K)=1.-(SEE(K)*(N-1))/(AUK(J,J)*(N-L-1))                        MR405
      SEE(K)=SQRT(ABS(SEE(K)/(N-L-1)))                                  MR410
      SBO(K)=SEE(K)/SQRT(N)                                             MR415
      TO(K)=BO(K)/SBO(K)                                                MR416
      DO 220 I=1,L                                                      MR420
      B(I,K)=CUB(I,J)                                                   MR425
      SB(I,K)=SEE(K)*SQRT(ABS(CUB(I,I)))                                MR430
  220 T(I,K)=B(I,K)/SB(I,K)                                             MR435
C     START OF PRINTOUT                                                 MR440
  210 DO 300 J=1,M                                                      MR450
      PRINT 310                                                         MR460
      PRINT 310
      PRINT 310
  310 FORMAT(1H0)
      IF (NSW.EQ.0) GO TO 320                                           MR480
      PRINT 330,J,BO(J),(I,B(I,J),I=1,L)                                MR490
  330 FORMA (T2,'SINCE THE NUMBER OF VARIABLES JUST EQUALS THE NUMBER OF MR500
     1 DATA, NO VARIANCES CAN BE COMPUTED.'//T2,'B( 0,',I2,') =',T12,E14 MR500
     2.8/(T2,'B(',I2,',',I2,') =',T12,E14.8))                           MR500
      GO TO 300                                                         MR510
  320 RSQ=SQRT(ABS(MR(J)))                                              MR511
      PRINT 340,J,MR(J),RSQ,SEE(J),BO(J),SBO(J),TO(J)       ,(I,B(I,J),S MR520
     1B(I,J),T(I,J)         ,I=1,L)                                     MR520
  340 FORMAT(T2,'DEPENDENT VARIABLE',T20,'MULTIPLE R-SQD EST.',T40,'MULT MR530
     2TIPLE R EST.',T60,'STD. ERROR OF EST.',/T4,I2,T20,3(G14.8,6X)///T2 MR530
     3,'INDEP. VARIABLE',T20,'WEIGHT',T40,'STD. DEV. OF WT.',T60,'STUDEN MR530
     4T T'/T4,'INTERCEPT',T20,3(G14.8,6X)/(T4,I2,T20,3(G14.8,6X)))      MR53
  300 CONTINUE                                                          MR540
      RETURN
      END
C FOR MEANS
      SUBROUTINE MEANS(X,Y,MAX,N,IROWS)                                 MNS010
C     COMPUTES THE MEANS OF (MAX)VARIABLES BASED ON (N) OBSERVATIONS API MNS020
C     ECE AND YIELDS DEVIATIONS FROM THE MEANS                          MNS020
      DIMENSION X(IROWS,N),Y(IROWS)                                     MNS030
      DO 100 I=1, MAX                                                   MNS040
```

```
      Y(I)=0.                                                           MNS050
      DO 110 K=1,N                                                      MNS060
  110 Y(I)=Y(I)+X(I,K)                                                  MNS070
      Y(I)=Y(I)/N                                                       MNS080
      DO 100 K=1,N                                                      MNS090
  100 X(I,K)=X(I,K)-Y(I)                                                MNS100
      RETURN                                                            MNS110
      END
* FOR SUMSQ
      SUBROUTINE SUMSQ(X,A,MAX,N,IROWS)                                 SMSQ010
C     FINDS THE SUM OF SQUARES OF (N) OBSERVATIONS OF (MAX)VARIABLES AND SMSQ020
C  STORES THEM IN A SEPARATE MATRIX                                     SMSQ025
      DIMENSION X(IROWS,N),A(IROWS,MAX)                                 SMSQ030
      DO 100 I=1,MAX                                                    SMSQ044
      DO 100 J=1,I                                                      SMSQ050
      A(I,J)=0.                                                         SMSQ060
      DO 110 K=1,N                                                      SMSQ070
  110 A(I,J)=A(I,J)+X(I,K)*X(J,K)                                       SMSQ080
  100 A(J,I)=A(I,J)                                                     SMSQ090
      RETURN                                                            SMSQ100
      END                                                                MR550
* FOR MXMLT
      SUBROUTINE MXMLT(A,B,C,M,L,N,MX,LX)
C.......................................................................
C       REAL MATRIX MULTIPLICATION.
C.......................................................................
C.......................................................................
      DIMENSION A(MX,L),B(LX,N),C(MX,N)
      DO 1 J=1,N
      DO 1 I=1,M
      C(I,J)=0.
      DO 1 K=1,L
    1 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
* FOR INV3
      SUBROUTINE INV3(B,A,N,DET )
C                                                                       INVR0030
C                                          GEORGE TOWNSEND              INVR0040
C                                          DEPT 190/600, GB95           INVR0050
C                                                                       INVR0060
C     SUBROUTINE FOR INVERTING SQUARE MATRICES WHICH ARE 6 BY 6 OR LESS INVR0070
C     (TO INVERT LARGER MATRICES,SAY M BY M, DIMENSION IPIVOT(M),A(M*M),INVR0080
C     INDEX(M,2),PIVOT(M) AND RECOMPILE)                                INVR0090
C     N IS THE SIZE OF A (A IS AN N BY N MATRIX)                        INVR0110
C     THE SUBROUTINE RETURNS A INVERSE IN PLACE OF A AND THE DETERMINANTINVR0120
C     IN DETERM.                                                        INVR0130
      DIMENSION IPIVOT(7),INDEX(7,2),PIVOT(7),A(1),B(1)
C     B IS THE SQUARE MATRIX TO BE INVERTED
C     A IS THE ADJUSTED INVERSE (N BY N )
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM),(AMAX,T,SWAP)            INVR0165
      M = N*N                                                           INVR0170
      DO 1 I=1,M                                                        INVR0180
    1 A(I)=B(I)                                                         INVR0190
      PIDIAG=1.0
      DO 2 I=1,N
    2 PIDIAG=PIDIAG*B(I)
      DETERM=1.0                                                        INVR0200
      DO 20  J=1,N                                                      INVR0210
```

```
   20 IPIVOT(J)=0
      DO 550  I=1,N
      AMAX=0.0
      DO 105 J=1,N
      IF(IPIVOT(J)-1) 60,105,60
   60 DO 100 K=1,N
      IF(IPIVOT(K)-1) 80,100,740
   80 M = N*(K-1)+J
      IF (ABS(AMAX)-ABS(A(M))) 85,100,100
   85 IROW=J
      ICOLUM=K
      AMAX = A(M)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
      IF(IROW-ICOLUM) 140,260,140
  140 DETERM=-DETERM
      DO 200 L=1,N
      M = N*(L-1)
      M1 = M+ICOLUM
      M = M+IROW
      SWAP = A(M)
      A(M) = A(M1)
  200 A(M1) = SWAP
  260 INDEX(I,1)=IROW
      INDEX(I,2)=ICOLUM
      M = N*(ICOLUM-1)+ICOLUM
      PIVOT(I) = A(M)
      DETERM=DETERM*PIVOT(I)
      A(M) = 1.0
      DO 350 L=1,N
      M = N*(L-1)+ICOLUM
  350 A(M) = A(M)/PIVOT(I)
      DO 550 L1=1,N
      IF(L1-ICOLUM) 400,550,400
  400 M = N*(ICOLUM-1)+L1
      T = A(M)
      A(M) = 0.
      DO 450  L=1,N
      M = N*(L-1)
      M1 = M+ICOLUM
      M = M+L1
  450 A(M) = A(M)-A(M1)*T
  550 CONTINUE
      DO 710 I=1,N
      L=N+1-I
      IF(INDEX(L,1)-INDEX(L,2)) 630,710,630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      M = N*(JROW-1)
      M1 = N*(JCOLUM-1)
      DO 705 K=1,N
      M = M+1
      M1 = M1+1
      SWAP = A(M)
      A(M) = A(M1)
      A(M1) = SWAP
  705 CONTINUE
  710 CONTINUE
```

```
  740 DET = DETERM                                                      INVR0810
      IF (DET-PIDIAG/1.E6) 3,3,4
    3 WRITE (6,5)
    5 FORMAT (1H1,' ERROR - MATRIX IS SINGULAR')
      RETURN
    4 CONTINUE
      CALL ADJUST( B,N,A,1 )
      M = N*N                                                           INVR0820
      RETURN                                                            INVR0850
      END                                                               [illegible]
.I FOR ADJT
      SUBROUTINE ADJUST( A,N,AINV,NUMBER )
.
.                                            G E TOWNSEND
.                                            NORTH AMERICAN ROCKWELL
C
C     THIS ROUTINE IS DESIGNED TO CORRECT THE NUMERICAL INVERSE OF A
C              MATRIX SO AS TO ASSURE THAT THE PRODUCT OF A AND AINV=I
C
      DIMENSION  A(N,N),AINV(N,N),ERROR(100),BINV(100)
C
      DO 1 K=1,NUMBER
      CALL MXMLT (A,AINV,ERROR,N,N,N,N,N)
      DO 10 I=1,N
      DO 10 J=1,N
      L = N*(J-1) + I
      ERROR(L) =-ERROR(L)
      IF( I.EQ. J )   ERROR( L ) = ERROR(L) + 2.
   10 CONTINUE
      CALL MXMLT(ERROR,AINV,BINV,N,N,N,N,N)
      DO 20 I=1,N
      DO 20 J=1,N
      L = N*(J-1) + I
   20 AINV(I,J) = BINV(L)
    1 CONTINUE
      RETURN
      END
. FOR DONA
      SUBROUTINE DONA
      PARAMETER IXXX=2,IYYY=10,IPSI=30,IOB=82,INDEP=7,MAX=16,IR=80
      COMMON/BLK2/PSI(IOB,IPSI),X(IOB,IXXX),YY(IOB,IYYY)
      COMMON/BLK3/S(INDEP,INDEP)
      COMMON     R(INDEP,1),T(INDEP,1),RR(INDEP,1,IOB),TT(INDEP,1,10)
C R AND RR MATRICES ARE INDEPENDENT VECTORS AND T AND TT MATRICES ARE
C DEPENDENT VECTORS.  RR MATRIX IS THE TIME EQUIVALENT OF THE R MATRIX
C AND THE TT MATRIX IS THE TIME EQUIVALENT OF THE T MATRIX.
      R(1,1)=PSI(1,1)
      R(2,1)=PSI(1,2)
      R(3,1)=PSI(1,5)
      R(4,1)=PSI(1,9)
      R(5,1)=PSI(1,10)
      R(6,1)=X(1,1)
      R(7,1)=X(1,2)
      DO 4999 I=1,INDEP
 4999 RR(I,1,1)=R(I,1)
      DO 5000 J=1,10
      CALL MXMLT (S,R,T,INDEP,INDEP,1,INDEP,INDEP)
      DO 5001 I=1,5
      R(I,1)=T(I,1)
```

```
      RR(I,1,J+1)=R(I,1)
5001  TT(I,1,J)=T(I,1)
      TT(6,1,J)=T(6,1)
      TT(7,1,J)=T(7,1)
      R(6,1)=X(J+1,1)
      R(7,1)=X(J+1,2)
      RR(6,1,J+1)=R(6,1)
      RR(7,1,J+1)=R(7,1)
5000  CONTINUE
      WRITE (6,9472)
9472  FORMAT(1H1,50X,'DONA INDEPENDENT VARIABLES')
      WRITE (6,1237)
1237  FORMAT (6X,5HPSI 1,8X,5HPSI 2,8X,5HPSI 3,8X,5HPSI 4,8X,5HPSI 5,9X,
     *2HX1,11X,2HX2)
      WRITE(6,9471)((RR(I,1,J),I=1,INDEP),J=1,IR)
9471  FORMAT(1H ,7G13.5)
      WRITE (6,9473)
9473  FORMAT(1H1,50X,' SIMCO AND DONA OUTPUT VA   _ES')
      WRITE (6,1238)
1238  FORMAT (6X,5HSIMCO,9X,4HDONA,29X,5HSIMCO,9X,4HDONA/ 8X,2HY1,11X,2H
     *Y1,31X,2HY2,11X,2HY2)
      WRITE (6,9476) (YY(J,1),TT(6,1,J),YY(J,2),TT(7,1,J),J=1,IR)
9476  FORMAT (1H ,2G13.5,20X,2G13.5)
      WRITE (6,9474)
9474  FORMAT(1H1,49X,' SIMCO AND DONA STATE VARIABLES')
      WRITE (6,1236)
1236  FORMAT (6X,5HSIMCO,8X,4HDONA,9X,5HSIMCO,8X,4HDONA,9X,5HSIMCO,8X,4H
     *DONA,9X,5HSIMCO,8X,4HDONA,9X,5HSIMCO,8X,4HDONA/ 6X,5HPSI 1,8X,5HPS
     *I 1,8X,5HPSI 2,8X,5HPSI 2,8X,5HPSI 3,8X,5HPSI 3,8X,5HPSI 4,8X,5HPS
     *I 4,8X,5HPSI 5,8X,5HPSI 5)
      WRITE (6,9477) (PSI(J,1),TT(1,1,J),PSI(J,2),TT(2,1,J),PSI(J,5),TT(
     *3,1,J),PSI(J,9),TT(4,1,J),PSI(J,10),TT(5,1,J),J=1,IR)
9477  FORMAT (1H ,10G13.5)
      RETURN
      END
* XQT MAIN
* FIN
* FIN
```

APPENDIX E

COMPUTER RESULTS OF THE LABORATORY SYSTEM

WEEK	AUC	ADDW1	SAL	WC	TUS1	COH	TUS	GIS	WGP	WNP	ULS	FC	ZT	Z	MT	SP
2	9.	67.00	576.00	2500.00	67.00	3043.15	9.	900.00	450.00	988.15	.0	36.	6.00	3.0	-1	50.00
3	50.	42.00	768.00	2500.00	42.00	3939.58	50.	5000.00	2500.00	1592.15	.0	36.	5.00	2.0	0	50.00
4	57.	40.00	384.00	2500.00	40.00	5570.00	57.	5700.00	2850.00	2224.85	.0	39.	5.00	2.0	0	50.00
5	33.	39.00	384.00	2500.00	39.00	6001.23	33.	3300.00	1650.00	976.42	.0	47.	5.00	2.0	1	50.00
6	35.	44.00	576.00	2500.00	44.00	4288.50	35.	3500.00	1750.00	1128.81	.0	42.	6.00	3.0	0	50.00
7	44.	50.00	576.00	2500.00	50.00	5070.88	44.	4400.00	2200.00	1874.62	.0	37.	6.00	3.0	-1	50.00
8	47.	60.00	768.00	2500.00	60.00	6160.58	47.	4700.00	2350.00	2340.31	.0	42.	5.00	2.0	1	50.00
9	46.	59.00	576.00	.00	59.00	8702.08	46.	4600.00	2300.00	2424.38	.0	46.	6.00	3.0	0	.00
10	59.	47.00	576.00	.00	47.00	12001.42	59.	5900.00	2950.00	2482.88	.0	51.	6.00	3.0	0	.00
11	68.	52.00	576.00	1050.00	52.00	15156.23	68.	6800.00	3400.00	3197.04	.0	58.	6.00	3.0	0	21.00
12	44.	50.00	576.00	4050.00	50.00	14256.69	44.	4400.00	2200.00	1903.81	.0	57.	6.00	3.0	0	81.00
13	70.	55.00	576.00	.00	55.00	18665.85	70.	7000.00	3500.00	3470.35	.0	61.	6.00	3.0	1	.00
14	35.	53.00	768.00	300.00	40.00	21493.46	35.	3500.00	1750.00	803.62	.0	50.	7.00	4.0	-1	6.00
15	65.	43.00	960.00	4600.00	.00	21769.96	65.	6500.00	3250.00	112.58	.0	57.	6.00	3.0	0	92.00
16	33.	64.00	576.00	.00	117.00	24243.19	33.	3300.00	1650.00	4692.85	.0	44.	6.00	3.0	0	.00
17	60.	8.00	576.00	.00	6.00	26623.62	5.	500.00	250.00	-2126.04	.0	53.	6.00	3.0	-1	.00
18	34.	38.00	576.00	11600.00	36.00	19815.39	89.	8900.00	4450.00	3335.85	.0	42.	6.00	3.0	0	*****
19		76.00		5850.00		18547.62		5500.00		3895.54		50.				

	55.		576.00		70.00		55.		2750.00		.0		5.00	3.0	1	*****
20	48.	41.00	570.00	6450.00	41.00	14716.39	48.	4800.00	2400.00	1604.58	.0	46.	6.00	3.0	0	*****
21	21.	60.00	576.00	.00	60.00	14593.23	21.	2100.00	1050.00	1211.08	.0	41.	6.00	3.0	0	.00
22	87.	80.00	576.00	2950.00	80.00	19011.54	87.	8700.00	4350.00	5512.23	.0	52.	6.00	3.0	1	59.00
23	7.	52.00	576.00	.00	52.00	20696.73	7.	700.00	350.00	-80.58	.0	38.	7.00	3.0	-1	.00
24	35.	46.00	768.00	1100.00	46.00	22140.73	35.	3500.00	1750.00	835.15	.0	43.	6.00	2.0	0	22.00
25	33.	44.00	576.00	.00	44.00	23434.47	33.	3300.00	1650.00	651.04	.0	25.	6.00	3.0	-1	.00
26	73.	45.00	768.00	4350.00	45.00	25474.89	73.	7300.00	3650.00	2681.04	.0	47.	5.00	2.0	1	87.00
27	64.	81.00	576.00	250.00	81.00	27371.47	64.	6400.00	3200.00	4410.27	.0	57.	6.00	3.0	0	5.00
28	15.	46.00	576.00	.00	46.00	28475.97	15.	1500.00	750.00	239.69	.0	51.	6.00	3.0	0	.00
29	64.	25.00	576.00	10260.00	25.00	21695.66	64.	6400.00	3200.00	1611.27	.0	48.	6.00	3.0	0	*****
30	62.	50.00	576.00	.00	50.00	27025.27	62.	6200.00	3100.00	2746.35	.0	47.	6.00	3.0	0	.00
31	63.	44.00	576.00	.00	44.00	31967.66	63.	6300.00	3150.00	2520.81	.0	63.	6.00	3.0	0	.00
32	69.	54.00	768.00	4450.00	54.00	30113.12	69.	6900.00	3450.00	2935.50	.0	65.	6.00	4.0	1	89.00
33	52.	55.00	768.00	.00	55.00	34317.70	52.	5200.00	2600.00	2349.77	.0	61.	7.00	4.0	0	.00
34	39.	46.00	768.00	7100.00	46.00	36655.89	39.	3900.00	1950.00	1310.54	.0	53.	7.00	4.0	-1	*****
35	58.	24.00	960.00	.00	24.00	34911.35	58.	5800.00	2900.00	883.92	.0	50.	6.00	3.0	0	.00
36	59.	[illegible]	576.00	.00	56.00	39912.35	59.	5900.00	2950.00	2934.31	.0	52.	6.00	3.0	0	.00
37	43.	[illegible]	576.00	.00	56.00	43275.47	43.	4300.00	2150.00	2157.04	.0	53.	6.00	3.0	0	.00
38	50.	47.00	576.00	.00	47.00	46347.55	50.	5000.00	2500.00	2075.31	.0	62.	6.00	3.0	1	.00

39	75.	44.00	768.00	2600.00	44.00	47676.39	52.	5200.00	2600.00	1840.88	.0	68.	7.00	4.0	0	52.00
40	30.	29.00	768.00	15150.00	29.00	38257.43	88.	8800.00	4400.00	2816.31	.0	63.	7.00	4.0	-1	*****
41	73.	69.00	768.00	.00	69.00	42453.16	73.	7300.00	3650.00	3083.31	.0	59.	6.00	4.0	0	.00
42	49.	38.00	960.00	.00	38.00	45618.63	49.	4900.00	2450.00	1161.19	.0	51.	6.00	3.0	0	.00
43	61.	50.00	576.00	3950.00	50.00	45912.40	61.	6100.00	3050.00	2743.58	.0	61.	6.00	3.0	1	73.00
44	35.	65.00	768.00	.00	65.00	50475.20	35.	3500.00	1750.00	2010.85	.0	48.	7.00	4.0	-1	.00
45	46.	58.00	960.00	4250.00	58.00	50390.47	46.	4600.00	2300.00	2004.08	.0	47.	6.00	3.0	0	85.00
46	44.	73.00	576.00	7050.00	73.00	46573.55	44.	4400.00	2200.00	3027.69	.0	42.	6.00	3.0	1	*****
47	2.	56.00	576.00	6900.00	56.00	41388.94	2.	200.00	100.00	-114.27	.0	31.	7.00	3.0	-1	*****
48	62.	65.00	768.00	1100.00	65.00	43513.01	62.	6200.00	3100.00	3127.81	.0	36.	6.00	2.0	0	22.00
49	85.	26.00	384.00	.00	26.00	48830.94	85.	8500.00	4250.00	2735.69	.0	50.	6.00	2.0	0	.00
50	41.	56.00	576.00	5650.00	56.00	45998.45	41.	4100.00	2050.00	1618.42	.0	63.	6.00	3.0	1	*****
51	75.	29.00	768.00	.00	29.00	48232.13	75.	7500.00	3750.00	2175.54	.0	67.	7.00	4.0	-1	.00
52	33.	33.00	768.00	.00	33.00	51374.59	33.	3300.00	1650.00	56.12	.0	50.	6.00	4.0	-1	.00
53	76.	51.00	960.00	.00	51.00	57633.63	76.	7600.00	3800.00	3307.27	.0	61.	5.00	3.0	1	.00
54	84.	51.00	768.00	.00	51.00	62299.98	84.	8400.00	4200.00	3920.77	.0	64.	6.00	4.0	1	.00
55	55.	41.00	768.00	.00	41.00	65829.17	27.	2700.00	1350.00	389.23	.0	72.	7.00	4.0	0	.00
56	31.	51.00	768.00	15500.00	51.00	53307.63	59.	5900.00	2950.00	2439.31	.0	57.	7.00	4.0	-1	*****
57	46.	48.00	960.00	.00	48.00	57603.44	46.	4600.00	2300.00	1471.00	.0	44.	6.00	3.0	0	.00

58	42.	28.00	576.00	7400.00	26.00	55065.94	42.	4200.00	2100.00	689.62	.0	40.	6.00	3.0	-1	*****
59	71.	56.00	768.00	.00	56.00	58347.75	71.	7100.00	3550.00	3316.54	.0	53.	5.00	2.0	1	.00
60	50.	24.00	576.00	.00	24.00	62381.83	50.	5000.00	2500.00	885.73	.0	54.	6.00	3.0	-1	.00
61	66.	44.00	576.00	.00	44.00	66576.21	66.	6600.00	3300.00	2885.58	.0	62.	5.00	3.0	2	.00
62	69.	52.00	768.00	10760.00	52.00	59798.60	69.	6900.00	3450.00	3011.65	.0	62.	7.00	4.0	-1	*****
63	80.	57.00	768.00	.00	57.00	65732.18	80.	8000.00	4800.00	3630.42	.0	72.	6.00	4.0	1	.00
64	0.	60.00	768.00	.00	60.00	66630.45	0.	.00	.00	-7.08	.0	50.	7.00	4.0	-1	.00
65	59.	17.00	960.00	4700.00	17.00	66372.48	59.	5900.00	2950.00	597.00	.0	46.	6.00	3.0	0	94.00
66	46.	47.00	576.00	.00	47.00	70710.41	46.	4600.00	2300.00	1834.69	.0	35.	6.00	3.0	-1	.00
67	33.	70.00	768.00	.00	70.00	74766.95	33.	3300.00	1650.00	2147.81	.0	46.	5.00	2.0	1	.00
68	55.	53.00	576.00	2050.00	53.00	74067.57	55.	5500.00	2750.00	2598.04	.0	45.	6.00	3.0	0	41.00
69	61.	41.00	576.00	.00	41.00	78674.53	61.	6100.00	3050.00	2316.81	.0	50.	6.00	3.0	0	.00
70	58.	58.00	576.00	20050.00	58.00	62365.95	58.	5800.00	2900.00	2957.54	.0	58.	6.00	3.0	0	*****
71	85.	47.00	576.00	.00	47.00	66817.61	85.	8500.00	4250.00	3725.31	.0	68.	6.00	3.0	1	.00
72	69.	6.00	768.00	.00	6.00	71389.61	69.	6900.00	3450.00	711.23	.0	71.	7.00	4.0	-1	.00
73	72.	46.00	768.00	.00	46.00	76423.80	72.	7200.00	3600.00	2674.92	.0	75.	6.00	4.0	0	.00
74	37.	51.00	768.00	.00	51.00	79570.99	37.	3700.00	1850.00	1583.77	.0	59.	6.00	4.0	-1	.00
75	37.	29.00	960.00	1850.00	29.00	82027.80	37.	3700.00	1850.00	276.85	.0	49.	5.00	3.0	1	37.00
76	24.	32.00	576.00	10100.00	32.00	75140.57	24.	2400.00	1200.00	-55.85	.0	33.	6.00	3.0	-2	*****
77		41.00		.00		79819.42		5400.00		1515.19		38.				

	54.		768.00		41.00		54.		2700.00		.0		4.00	2.0	0	.00
78		87.00		.00		83894.04		6900.00		9342.77		49.				
	69.		384.00		87.00		69.		3450.00		.0		4.00	2.0	2	.00
79		42.00		8500.00		79812.46		8000.00		1262.54		54.				
	40.		576.00		42.00		40.		2800.00		.0		6.00	3.0	0	*****
80		45.00		5850.00		74765.65		2100.00		455.04		43.				
	21.		576.00		45.00		21.		1050.00		.0		6.00	3.0	0	*****
81		61.00		3850.00		73858.69		3400.00		1982.92		32.				
	34.		576.00		61.00		34.		1700.00		.0		6.00	3.0	-1	77.00
82		55.00		.00		78317.62		5300.00		2370.00		36.				
	53.		768.00		55.00		53.		2650.00		.0		5.00	2.0	0	.00

TIME

---- DEL PSI, BACKWARD DIFFERENCE OF STATE VARIABLES -----

	2	3	4	5	6	7	8	9	10	11	12	13
TUS	-41.00	41.00	7.00	-24.00	2.00	9.00	3.00	-1.00	13.00	9.00	-24.00	26.00
FD	-13.67	.00	2.33	8.00	-5.00	-4.33	4.67	3.67	5.00	7.00	-.67	3.67

----- DEL X, BACKWARD DIFFERENCE OF INPUTS -----

	2	3	4	5	6	7	8	9	10	11	12	13
ADDWS	-41.00	41.00	7.00	-24.00	2.00	9.00	3.00	-1.00	13.00	9.00	-24.00	26.00
ADDWS1	27.00	-25.00	-2.00	-1.00	5.00	6.00	10.00	-1.00	-12.00	5.00	-2.00	5.00

----- DELTA PSI, FORWARD DIFFERENCE OF STATE VARIABLES -----

	2	3	4	5	6	7	8	9	10	11	12	13
TUS	41.00	7.00	-24.00	2.00	9.00	3.00	-1.00	13.00	9.00	-24.00	26.00	-35.00
FD	.00	2.33	8.00	-5.00	-4.33	4.67	3.67	5.00	7.00	-.67	3.67	-11.00

----- DEL Y, BACKWARD DIFFERENCE OF OUTPUTS -----

	2	3	4	5	6	7	8	9	10	11	12	13
SAL	76.00	192.00	-384.00	.00	192.00	.00	192.00	-192.00	.00	.00	.00	.00
WC	-500.00	.00	.00	.00	.00	.00	.00	-2500.00	.00	1050.00	3000.00	-4050.00

X MATRIX

-50.00000	85.00000	41.00000	-13.66667	5.666666	-41.00000	27.00000	.0000000
.0000000	-135.0000	.0000000	.0000000	-2.666667	41.00000	-25.00000	21.00000
21.00000	1.000000	-7.000000	2.333333	-3.333334	7.000000	-2.000000	60.00000
60.00000	-1.000000	17.00000	8.000000	-9.333334	-24.00000	-1.000000	-81.00000
-81.00000	9.000000	15.00000	-5.000000	.6666670	2.000000	5.000000	6.000000
6.000000	77.00000	6.000000	-4.333334	3.333333	9.000000	6.000000	56.00000
56.00000	47.00000	3.000000	4.666667	7.000000	3.000000	10.00000	-14.00000
-14.00000	-30.00000	-45.00000	3.666667	5.000000	-1.000000	-1.000000	39.00000
39.00000	-77.00000	-59.00000	5.000000	-1.000000	13.00000	-12.00000	23.00000
23.00000	7.000000	-47.00000	7.000000	-2.666667	9.000000	5.000000	-126.0000
-126.0000	-12.00000	37.00000	-.6666665	-3.000000	-24.00000	-2.000000	115.0000
115.0000	63.00000	-70.00000	3.666667	2.666667	26.00000	5.000000	-129.0000
-129.0000	-56.00000	-29.00000	-11.00000	.3333335	-35.00000	-2.000000	59.00000
59.00000	-28.00000	27.00000	7.000000	-2.333333	30.00000	-10.00000	-59.00000
-59.00000	200.0000	-33.00000	-12.33333	3.000000	-32.00000	21.00000	.0000000
.0000000	-200.0000	-5.000000	8.333334	-15.00000	27.00000	-56.00000	22.00000
22.00000	.0000000	143.0000	-10.33333	-1.666667	-26.00000	38.00000	65.00000
65.00000	.0000000	62.00000	7.333333	4.000000	21.00000	38.00000	-82.00000
-82.00000	72.00000	61.00000	-4.000000	11.00000	-7.000000	-35.00000	-5.000000
-5.000000	73.00000	-21.00000	-4.333334	7.333333	-27.00000	19.00000	204.0000
204.0000	-33.00000	-28.00000	10.66667	1.333333	66.00000	20.00000	-204.0000
-204.0000	-55.00000	-7.000000	-13.66667	3.666667	-80.00000	-28.00000	.0000000
.0000000	-57.00000	-13.00000	4.666667	-4.666667	28.00000	-6.000000	.0000000
.0000000	.0000000	-33.00000	-18.00000	-12.00000	-2.000000	-2.000000	89.00000
89.00000	.0000000	14.00000	22.00000	-2.333333	40.00000	1.000000	53.00000
53.00000	179.0000	-59.00000	9.666667	11.66667	-9.000000	36.00000	-142.0000
-142.0000	-162.0000	-15.00000	-6.000000	.6666665	-49.00000	-35.00000	.0000000
.0000000	-17.00000	140.0000	-3.000000	-6.666666	49.00000	-21.00000	52.00000
52.00000	.0000000	-62.00000	-.6666665	-10.33333	-2.000000	25.00000	156.0000
156.0000	.0000000	-63.00000	16.00000	-.6666665	1.000000	-6.000000	-118.0000
-118.0000	109.0000	20.00000	1.666666	9.666667	6.000000	18.00000	-85.00000
-85.00000	-38.00000	-52.00000	-3.333333	1.666667	-17.00000	1.000000	-5.000000
-5.000000	-18.00000	103.0000	-8.000000	1.333333	-13.00000	-7.000000	.0000000
.0000000	-53.00000	-58.00000	-3.666667	-10.00000	19.00000	-29.00000	79.00000
79.00000	1.000000	-59.00000	2.333333	.3333335	1.000000	32.00000	-40.00000
-40.00000	79.00000	-43.00000	1.333333	2.666667	-16.00000	.0000000	99.00000
99.00000	27.00000	-50.00000	9.000000	7.666667	47.00000	-9.000000	-36.00000
-36.00000	-102.0000	.0000000	5.333333	-4.000000	-10.00000	-3.000000	-17.00000
-17.00000	-5.000000	215.0000	-4.333333	-9.000000	-45.00000	-15.00000	-63.00000
-63.00000	108.0000	-73.00000	-4.000000	7.333333	43.00000	40.00000	-22.00000
-22.00000	-108.0000	-49.00000	-8.666667	-2.000000	-24.00000	-31.00000	113.0000
113.0000	114.0000	18.00000	10.33333	7.000000	12.00000	12.00000	-113.0000
-113.0000	-46.00000	-35.00000	-12.66667	-1.333333	-26.00000	15.00000	.0000000
.0000000	53.00000	39.00000	-1.000000	6.666666	11.00000	-7.000000	.0000000
.0000000	46.00000	97.00000	-5.666666	7.666667	-2.000000	15.00000	.0000000
.0000000	-145.0000	136.0000	-11.00000	-3.000000	-42.00000	-17.00000	.0000000
.0000000	83.00000	-40.00000	5.333333	2.333333	60.00000	9.000000	196.0000
196.0000	-95.00000	-85.00000	13.66667	-15.66667	23.00000	-39.00000	-82.00000
-82.00000	.0000000	72.00000	13.00000	.0000000	-44.00000	30.00000	34.00000
34.00000	.0000000	-75.00000	4.333334	-12.00000	34.00000	-27.00000	-148.0000
-148.0000	.0000000	-37.00000	-17.33333	2.333333	-42.00000	4.000000	59.00000
59.00000	.0000000	-76.00000	11.66667	-1.666667	43.00000	18.00000	60.00000
60.00000	55.00000	-84.00000	9.000000	7.333333	8.000000	.0000000	-83.00000
-83.00000	.0000000	-27.00000	7.333333	2.666667	-29.00000	-10.00000	-36.00000
-36.00000	6.000000	251.0000	-15.00000	.0000000	-24.00000	10.00000	.0000000
.0000000	-25.00000	-96.00000	-12.66667	-1.000000	15.00000	-3.000000	.0000000
.0000000	-36.00000	166.0000	-4.333334	-4.333334	-4.000000	-20.00000	94.00000

94.00000	68.00000	-71.00000	13.33333	1.666667	29.00000	28.00000	-53.00000
-53.00000	-68.00000	-50.00000	1.333333	-8.000000	-21.00000	-32.00000	113.0000
113.0000	32.00000	-66.00000	8.000000	5.333333	16.00000	20.00000	-86.00000
-86.00000	16.00000	145.0000	-.6666665	-1.333333	3.000000	8.000000	115.0000
115.0000	113.0000	-80.00000	10.00000	11.00000	11.00000	5.000000	-161.0000
-161.0000	-50.00000	.0000000	-22.00000	5.333333	-80.00000	3.000000	.0000000
.0000000	-111.0000	35.00000	-3.333334	-11.66667	59.00000	-43.00000	.0000000
.0000000	.0000000	-46.00000	-11.33333	-3.333334	-13.00000	30.00000	.0000000
.0000000	39.00000	-33.00000	11.00000	3.333334	-13.00000	23.00000	.0000000
.0000000	105.0000	-14.00000	-1.333333	12.00000	22.00000	-17.00000	37.00000
37.00000	-133.0000	-61.00000	5.000000	-2.000000	6.000000	-12.00000	93.00000
93.00000	28.00000	343.0000	8.333334	-4.000000	-3.000000	17.00000	72.00000
72.00000	-21.00000	-85.00000	10.00000	-2.000000	27.00000	-11.00000	-125.0000
-125.0000	-18.00000	-69.00000	2.666666	-11.66667	-16.00000	-41.00000	40.00000
40.00000	.0000000	-72.00000	4.666667	-4.000000	3.000000	40.00000	-117.0000
-117.0000	.0000000	-37.00000	-16.00000	1.333333	-35.00000	5.000000	.0000000
.0000000	54.00000	.0000000	-10.66667	7.666667	.0000000	-22.00000	.0000000
.0000000	-54.00000	178.0000	-16.00000	-4.666667	-13.00000	3.000000	.0000000
.0000000	13.00000	-54.00000	5.666666	-3.333333	30.00000	9.000000	.0000000
.0000000	294.0000	-69.00000	10.66667	19.33333	15.00000	46.00000	40.00000
40.00000	-280.0000	90.00000	5.333333	3.333334	-29.00000	-45.00000	-40.00000
-40.00000	33.00000	96.00000	-11.00000	1.333333	-19.00000	3.000000	.0000000
.0000000	-60.00000	43.00000	-11.66667	-8.666667	13.00000	16.00000	.0000000

-135.0000	.0000000	.0000000	-2.666667	76.00000	-500.0000
1.000000	-7.000000	2.333333	-3.333334	192.0000	.0000000
-1.000000	17.00000	8.000000	-9.333334	-384.0000	.0000000
9.000000	15.00000	-5.000000	.6666670	.0000000	.0000000
77.00000	6.000000	-4.333334	3.333333	192.0000	.0000000
47.00000	3.000000	4.666667	7.000000	.0000000	.0000000
-30.00000	-46.00000	3.666667	5.000000	192.0000	.0000000
-77.00000	-59.00000	5.000000	-1.000000	-192.0000	-2500.000
7.000000	-47.00000	7.000000	-2.666667	.0000000	.0000000
-12.00000	37.00000	-.6666665	-3.000000	.0000000	1050.000
63.00000	-70.00000	3.666667	2.666667	.0000000	3000.000
-56.00000	-29.00000	-11.00000	.3333335	.0000000	-4050.000
-28.00000	27.00000	7.000000	-2.333333	192.0000	300.0000
200.0000	-33.00000	-12.33333	3.000000	192.0000	4300.000
-200.0000	-5.000000	8.333334	-15.00000	-384.0000	-4600.000
.0000000	143.0000	-10.33333	-1.666667	.0000000	.0000000
.0000000	62.00000	7.333333	4.000000	.0000000	11600.00
72.00000	81.00000	-4.000000	11.00000	.0000000	-5750.000
73.00000	-21.00000	-4.333334	7.333333	.0000000	600.0000
-33.00000	-28.00000	10.66667	1.333333	.0000000	-6450.000
-55.00000	-7.000000	-13.66667	3.666667	.0000000	2950.000
-57.00000	-13.00000	4.666667	-4.666667	.0000000	-2950.000
.0000000	-33.00000	-18.00000	-12.00000	192.0000	1100.000
.0000000	14.00000	22.00000	-2.333333	-192.0000	-1100.000
179.0000	-59.00000	9.666667	11.66667	192.0000	4350.000
-162.0000	-15.00000	-6.000000	.6666665	-192.0000	-4100.000
-17.00000	140.0000	-3.000000	-6.666666	.0000000	-250.0000
.0000000	-62.00000	-.6666665	-10.33333	.0000000	10200.00
.0000000	-63.00000	16.00000	-.6666665	.0000000	-10200.00
109.0000	20.00000	1.666666	9.666667	.0000000	.0000000
-38.00000	-52.00000	-3.333333	1.666667	192.0000	4450.000
-18.00000	103.0000	-8.000000	1.333333	.0000000	-4450.000
-53.00000	-58.00000	-3.666667	-10.00000	.0000000	7100.000
1.000000	-59.00000	2.333333	.3333335	192.0000	-7100.000
79.00000	-43.00000	1.333333	2.666667	-384.0000	.0000000
27.00000	-50.00000	9.000000	7.666667	.0000000	.0000000
-102.0000	.0000000	5.333333	-4.000000	.0000000	.0000000
-5.000000	215.0000	-4.333333	-9.000000	192.0000	2600.000
108.0000	-73.00000	-4.000000	7.333333	.0000000	12550.00
-108.0000	-49.00000	-8.666667	-2.000000	.0000000	-15150.00
114.0000	18.00000	10.33333	7.000000	192.0000	.0000000
-46.00000	-35.00000	-12.66667	-1.333333	-384.0000	3950.000
43.00000	39.00000	-1.000000	6.666666	192.0000	-3950.000
46.00000	97.00000	-5.666666	7.666667	192.0000	4250.000
-145.0000	136.0000	-11.00000	-3.000000	-384.0000	2800.000
63.00000	-40.00000	5.333333	2.333333	.0000000	-150.0000
-95.00000	-85.00000	13.66667	-15.66667	192.0000	-5800.000
.0000000	72.00000	13.00000	.0000000	-384.0000	-1100.000
.0000000	-75.00000	4.333334	-12.00000	192.0000	5650.000
.0000000	-33.00000	-17.33333	2.333333	192.0000	-5650.000
.0000000	-76.00000	11.66667	-1.666667	.0000000	.0000000
55.00000	-24.00000	3.000000	7.333333	192.0000	.0000000
.0000000	-27.00000	7.333333	2.666667	-192.0000	.0000000
6.000000	251.0000	-15.00000	.0000000	.0000000	.0000000
-25.00000	-46.00000	-12.66667	-1.000000	.0000000	15500.00
-36.00000	106.0000	-4.333334	-4.333334	192.0000	-15500.00
68.00000	-71.00000	13.33333	1.666667	-384.0000	7400.000

[illegible]	[illegible]	1.333333	-6.000000	192.0000	-7800.000
32.00000	[illegible]	6.000000	4.333333	-192.0000	.0000000
16.00000	145.0000	-.666665	-1.333333	.0000000	.0000000
[illegible]	[illegible]	10.00000	11.00000	192.0000	10700.00
-50.00000	.0000000	-22.00000	4.333333	.0000000	-10700.00
-111.0000	34.00000	-3.333334	-11.66667	.0000000	.0000000
.0000000	-46.00000	-11.33333	-3.333334	192.0000	4700.000
39.00000	-43.00000	11.00000	3.333334	-384.0000	-4700.000
105.0000	-14.00000	-1.333333	12.00000	192.0000	.0000000
-133.0000	-61.00000	5.000000	-2.000000	-192.0000	2050.000
28.00000	383.0000	8.333334	-4.000000	.0000000	-2050.000
-21.00000	-85.00000	10.00000	-7.000000	.0000000	20050.00
-18.00000	-69.00000	2.666666	-11.66667	.0000000	-20050.00
.0000000	-72.00000	4.666667	-6.000000	192.0000	.0000000
.0000000	-37.00000	-16.00000	1.333333	.0000000	.0000000
54.00000	.0000000	-10.66667	7.666667	.0000000	.0000000
-54.00000	178.0000	-16.00000	-4.666667	192.0000	1850.000
13.00000	-54.00000	5.666666	-3.333333	-384.0000	8250.000
254.0000	-69.00000	10.66667	10.33333	192.0000	-10100.00
-280.0000	90.00000	5.333333	3.333334	-384.0000	.0000000
33.00000	96.00000	-11.00000	1.333333	192.0000	6500.000
-60.00000	43.00000	-11.66667	-8.666667	.0000000	-650.0000
67.00000	-53.00000	4.333334	4.333334	.0000000	-2900.000

AIK MATRIX

509394.7	42344.75	-61507.49	37230.87	-2966.416	116714.0	19989.13	-212453.0
42344.75	568512.8	-69108.50	6349.209	27025.91	22121.00	91685.12	-51173.00
-61507.49	-69108.50	545597.7	-16608.25	-3984.300	-46305.20	-5402.550	89428.00
37230.87	6349.209	-16608.25	6087.686	124.2917	11656.67	964.4792	-18999.33
-2966.416	27025.91	-3984.300	124.2917	3500.660	-1513.133	4773.841	-6146.333
116714.0	22121.00	-46305.20	11656.67	-1513.133	69376.78	681.2000	-32302.00
19989.13	91685.12	-5402.550	964.4792	4773.841	681.2000	38623.48	-13001.00
-212453.0	-51173.00	89828.00	-18999.33	-6146.333	-32302.00	-13001.00	566926.0
29521.00	-251401.0	72035.40	7597.333	-19981.73	9665.399	-32828.40	46626.00
-5136.250	-123481.2	-78805.50	294.2084	5283.917	-24440.00	-20323.87	-59635.00
-3490.876	740.4581	-2127.816	-644.8543	-864.0027	-2314.400	922.7542	36559.00
6381.083	2479.417	-619.9000	843.7639	155.9834	794.9334	1215.525	-2682.666
-210920.5	-323924.5	-16737.80	12546.75	-10576.70	48931.20	-56143.95	74688.00
-1837269.	-5650019.	.313889E+06	-280805.2	-291926.2	-2684830.	-1325423.	5307550.

29521.00	-5136.250	-3490.876	6381.083	-210920.5	-1837269.
-251401.0	-123481.2	740.4581	2479.417	-323924.5	-5650019.
72035.40	-78805.50	-2127.816	-619.9000	-16737.80	.313889E+06
7597.333	294.2084	-644.8543	843.7639	12546.75	-280805.2
-19981.73	5283.917	-864.0027	155.9834	-10576.70	-291926.2
9665.399	-24440.00	-2314.400	794.9334	48931.20	-2684830.
-32828.40	-20323.87	922.7542	1215.525	-56143.95	-1325423.
46626.00	-59635.00	36559.00	-2682.666	74688.00	5307550.
568959.1	-77348.00	7899.133	26920.46	410841.6	5678611.
-77348.00	547282.6	-16341.87	-4443.917	-9588.502	-2760469.
7899.133	-16341.87	6783.886	220.6584	-20439.68	-61746.42
26920.46	-4443.917	220.6584	3487.283	11255.23	186354.6
410841.6	-9588.502	-20439.68	11255.23	2659911.	-4474008.
5678611.	-2760469.	-61746.42	186354.6	-4474008.	.313181E+10

AAA MATRIX

504394.7	42344.75	-61507.49	37230.87	-2906.416	116714.0	19989.13	-212453.0
42344.75	566612.8	-69188.50	6349.209	27025.91	22121.00	91685.12	-51173.00
-61507.49	-69188.50	545597.7	-16606.25	-3984.300	-86305.20	-5402.550	69828.00
37230.87	6349.209	-16606.25	6047.686	124.2017	11656.87	964.4792	-18999.33
-2906.416	27025.91	-3984.300	124.2017	3500.460	-1513.133	4773.841	-6146.333
116714.0	22121.00	-86305.20	11656.87	-1513.133	69376.78	681.2000	-32302.00
19989.13	91685.12	-5402.550	964.4792	4773.841	681.2000	38623.40	-13001.00
-212453.0	-51173.00	69828.00	-18999.33	-6146.333	-32302.00	-13001.00	406926.0
29521.00	-251401.0	72035.40	7597.333	-19981.73	9665.399	-32828.40	46626.00
-5136.250	-123401.2	-78865.50	294.2084	5283.917	-24480.00	-20323.67	-59635.00
-3490.876	740.4561	-2127.816	-644.6543	-864.0027	-2314.800	922.7542	36559.00
6381.093	2479.417	-619.9000	843.7639	155.9838	794.9334	1215.528	-2682.666
-210920.5	-323924.5	-16737.80	12546.75	-10576.70	48931.20	-56143.95	74688.00
-1837269.	-5650019.	.3138898+08	-280805.2	-291926.2	-2684830.	-1325423.	5307550.

29521.00	-5136.250	-3490.876	6381.093	-210920.5	-1837269.
-251401.0	-123401.2	740.4561	2479.417	-323924.5	-5650019.
72035.40	-78865.50	-2127.816	-619.9000	-16737.80	.3138898+08
7597.333	294.2084	-644.6543	843.7639	12546.75	-280805.2
-19981.73	5283.917	-864.0027	155.9838	-10576.70	-291926.2
9665.399	-24480.00	-2314.800	794.9334	48931.20	-2684830.
-32828.40	-20323.67	922.7542	1215.528	-56143.95	-1325423.
46626.00	-59635.00	36559.00	-2682.666	74688.00	5307550.
560959.1	-77340.00	7899.133	26920.46	810841.6	5678611.
-77340.00	547262.8	-16341.87	-4443.917	-9588.502	-2760469.
7899.133	-16341.87	6783.886	220.6584	-20439.68	-61746.42
26920.46	-4443.917	220.6584	3887.263	11255.23	186354.6
810841.6	-9588.502	-20439.68	11255.23	2659911.	-4474008.
5678611.	-2760469.	-61746.42	186354.6	-4474008.	.3131813+10

INTERCORRELATION MATRIX

1.000000	.7867995-01	-.1166714	.6256293	-.7024734-01	.6208527	.1425085	-.4180838	.5483568-01	-.9727764-02	-.5930378-01	.1513992	-.1811999	-.4599891-01
.7867995-01	1.000000	-.1242194	.1010162	.6057551	.1113754	.6186782	-.9531479-01	-.4419959	-.2213537	.1192210-01	.5567980-01	-.2633916	-.1338866
-.1166714	-.1242194	1.000000	-.2697537	-.9116765-01	-.2380052	-.3721661-01	.1708060	.1292913	-.1443261	-.3497507-01	-.1421156-01	-.1389404-01	.7593517
.6256293	.1010162	-.2697537	1.000000	.2520270-01	.5309421	.5887719-01	-.3201444	.1200372	.4771216-02	-.9392054-01	.1714183	.9229487-01	-.6019870-01
-.7024734-01	.6057551	-.9116765-01	.2520270-01	1.000000	-.9709461-01	.4105512	-.1459045	-.4477315	.1207189	-.1772968	.4464365-01	-.1096078	-.8816582-01
.6208527	.1113754	-.2380052	.5309421	-.9709461-01	1.000000	.1315958-01	-.1722464	.4864881-01	-.1254262	-.1066822	.5110696-01	.1139056	-.1821427
.1425085	.6186782	-.3721661-01	.5887719-01	.4105512	.1315958-01	1.000000	-.9291349-01	-.2214540	-.1397896	.5700606-01	.1047357	-.1751635	-.1205122
-.4180838	-.9531479-01	.1708060	-.3201444	-.1459045	-.1722464	-.9291349-01	1.000000	.8681911-01	-.1132200	.6234228	-.6380443-01	.6431988-01	.1332060
.5483568-01	-.4419959	.1292913	.1208372	-.4477315	.4864881-01	-.2214540	.8681911-01	1.000000	-.1386126	.1271451	.6043638	.3339645	.1345252
-.9727764-02	-.2213537	-.1443261	.4771216-02	.1207189	-.1254262	-.1397896	-.1132200	-.1386126	1.000000	-.2681967	-.1017224	-.7947148-02	-.6667746-01
-.5930378-01	.1192210-01	-.3497507-01	-.9392054-01	-.1772968	-.1066822	.5700606-01	.6234228	.1271451	-.2681967	1.000000	.4536674-01	-.2191595	-.1339598-01
.1513992	.5567980-01	-.1421156-01	.1714183	.4464365-01	.5110696-01	.1047357	-.6380443-01	.6043638	-.1017224	.4536674-01	1.000000	.1168631	.5638055-01
-.1811999	-.2633916	-.1389404-01	.9229487-01	-.1096078	.1139056	-.1751635	.6431988-01	.3339645	-.7947148-02	-.2191595	.1168631	1.000000	-.4901309-01
-.4599891-01	-.1338866	.7593517	-.6019870-01	-.8816582-01	-.1821427	-.1265122	.1332060	.1345252	-.6667746-01	-.1339598-01	.5638055-01	-.4901309-01	1.000000

DETERMINANT OF A = .10806021

IMPROVED INVERSE MATRIX A (INTERCORRELATED FORM)

2.1755	.17263	-.19375	-.93074	.13598	-.90319	-.41318
.17263	2.2610	.72076-01	-.11204-01	-.97464	-.41718	-1.0144
-.19375	.72076-01	1.1264	.29485	.70415-01	.23095	-.24369-01
-.93074	-.11204-01	.29485	1.8448	-.15754	-.34696	.11117
.13598	-.97464	.70415-01	-.15754	1.6783	.28877	-.97336-01
-.90319	-.41718	.23095	-.34696	.28877	1.8708	.27266
-.41318	-1.0144	-.24369-01	.11117	-.97336-01	.27266	1.7154

IMPROVED INVERSE MATRIX A (NO INTERCORRELATIONS)

.42707-05	.32077-06	-.36752-06	-.15645-04	.32201-05	-.48044-05	-.29457-05
.32077-06	.39763-05	.12940-06	-.17826-06	-.21845-04	-.21004-05	-.68453-05
-.36752-06	.12940-06	.20645-05	.47891-05	.16112-05	.11871-05	-.16787-06
-.15645-04	-.17826-06	.47891-05	.26553-03	-.31944-04	-.15803-04	.67865-05
.32201-05	-.21845-04	.16112-05	-.31944-04	.47943-03	.18530-04	-.83709-05
-.48044-05	-.21004-05	.11871-05	-.15803-04	.18530-04	.26966-04	.52673-05
-.29457-05	-.68453-05	-.16787-06	.67865-05	-.83709-05	.52673-05	.44414-04

K MATRIX

-.21245+06	24521.	-5136.2	-3490.9	6381.1	-.21092+06	-.18373+07
-51173.	-.25140+06	-.12348+06	740.46	2479.4	-.32392+06	-.56500+07
89828.	72035.	-78866.	-2127.8	-619.90	-16738.	.31389+08
-18949.	7597.3	294.21	-644.85	843.76	12547.	-.28081+06
-6140.3	-19982.	5283.9	-864.00	155.98	-10577.	-.39193+06
-32302.	9665.4	-24440.	-2314.4	794.93	48931.	-.26846+07
-13001.	-32828.	-20324.	922.75	1215.5	-56144.	-.13254+07

UW MATRIX

-.48881	-.11398	.15714	.18190-02	.81765-02	-1.2988	-.93843
.34497-01	-.34128	-.42788	.19083-01	-.17228-02	-.84748	2.1099
.11986	.12651	-.19288	-.10397-01	.17277-02	.11160	59.966
-.66304	2.2080	-.11768	-.86495-01	.11150	5.7925	148.27
-2.2810	-3.8656	4.7490	-.47809	.17770-01	2.2752	-1.9578
.48165	.69132-01	-.48843	-.50670-01	-.19206-01	2.3034	-22.397
.13696	.43357	-.19999	.37220-01	.26979-01	.77934	-33.652

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
1	.18836739	.40868390	73.182359

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	-.81761236	8.1820367	-.99927731-01
1	-.48880864	.15123630	-3.2122283
2	.34496570-01	.14593044	.23639050
3	.11988128	.10815017	1.1398105
4	-.66304468	1.1925062	-.55600937
5	-2.2810489	1.6025990	-1.4047992
6	.48184810	.38002964	1.2673912
7	.13849089	.48771468	.27881245

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
2	.22876276	.47619613	79.624830

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	-.95817029	8.3433098	-.11484295
1	-.11398022	.15421727	-.73908859
2	-.34127874	.14880683	-2.2934347
3	.12660650	.10722275	1.1798476
4	2.2079882	1.2160113	1.8157620
5	-3.8656111	1.6339833	-2.2473590
6	.69131670-01	.38752016	.17839[illegible]09
7	.43357052	.49732772	.87180043

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
3	.12004965	.34648167	78.076690

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	2.8884637	8.7292404	.32745[illegible]

1	.15713972	.16135079	.97390112
2	-.42787535	.15569007	-2.7482507
3	-.19256394	.11218248	-1.7167025
4	-.11768350	1.2722595	-.92499605-01
5	4.7960467	1.7095653	2.8048339
6	-.48242988	.40544542	-1.1898763
7	-.19998862	.52033225	-.38434792

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
4	.44224262-02	.66501324-01	9.2462000

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	.89933045-02	1.0337566	.86996345-02
1	.18189667-02	.19107899-01	.95194491-01
2	.19083282-01	.18437532-01	1.0350237
3	-.10396637-01	.13285162-01	-.78257506
4	-.56495351-01	.15066679	-.37496884
5	-.47508715	.20245465	-2.3466361
6	-.50678854-01	.48014701-01	-1.0554862
7	.37220234-01	.61620126-01	.60402725

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
5	-.46518743-01	.21568204	6.7967849

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	-.12309135-01	.75990366	-.16198283-01
1	.81764863-02	.14046016-01	.58212136
2	-.17227643-02	.13553236-01	-.12711092
3	.17276547-02	.97657838-02	.17690896
4	.11150124	.11075358	1.0067507
5	.17770262-01	.14882723	.11940597
6	-.19206078-01	.35295105-01	-.54415699
7	.26928696-01	.45296311-01	.59450085

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
6	.17672744	.42038963	166.49142

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	.81503144	18.614306	.43785217-01
1	-1.2985895	.34406579	-3.7742477
2	-.84747836	.33199484	-2.5526854
3	.11160295	.23921887	.46653071
4	5.7925487	2.7129769	2.1351265
5	2.2751629	3.6454916	.62410317
6	2.3033576	.86457524	2.6641495
7	.77933717	1.1095609	.70238340

DEPENDENT VARIABLE	MULTIPLE R-SQU EST.	MULTIPLE R EST.	STD. ERROR OF EST.
7	.58084635	.76213276	4076.3456

INDEP. VARIABLE	WEIGHT	STD. DEV. OF WT.	STUDENT T
INTERCEPT	-163.17800	455.74930	-.35804335
1	-.93843454	8.4240444	-.11139952
2	2.1458723	8.1285013	.26399359
3	59.965746	5.8649913	10.238319
4	148.27456	66.424032	2.2322428
5	-1.5577933	89.255554	-.17453180-01
6	-22.397477	21.168103	-1.0580767
7	-33.681942	27.160208	-1.2387391

S MATRIX

-.48681	.34497-01	.11985	-.66304	-2.2510	.48165	.13598
-.11394	-.34128	.12651	2.2080	-3.6656	.69132-01	.43357
.15714	-.42788	-.19258	-.11768	4.7950	-.43243	-.19999
.18190-02	.19083-01	-.10397-01	-.56495-01	-.47509	-.50679-01	.37220-01
.81766-02	-.17228-02	.17277-02	.11180	.17770-01	-.19206-01	.26929-01
-1.2946	-.84748	.11160	5.7925	2.2752	2.3034	.77934
-.93843	2.1489	59.966	144.27	-1.5578	-22.397	-33.652

DONA INDEPENDENT VARIABLES

PSI 1	PSI 2	PSI 3	PSI 4	PSI 5	X1	X2
50.000	50.000	160.00	50.000	50.000	50.000	40.000
-120.77	-55.869	159.32	-28.139	7.1623	9.0000	67.000
91.019	-5.7248	-5.8424	-2.7176	-1.9948	50.000	42.000
-9.4181	13.727	-23.763	.24907	.58276	57.000	40.000
33.617	13.079	-38.510	-1.1089	-.12118	33.000	39.000
2.1293	4.1999	-17.635	.58443	.47157	35.000	44.000
18.389	17.152	-21.558	-.12576	.56590	44.000	50.000
15.876	11.692	-28.798	-.45715-01	.58096	47.000	60.000
18.758	17.591	-28.645	.12925	.77818	46.000	59.000
16.378	14.403	-29.299	.18746	.80679	59.000	47.000
21.917	11.358	-31.459	-1.0236	.22287	68.000	52.000
25.914	13.752	-37.277	-.96987	.88521-01	44.000	50.000
11.853	9.8916	-25.321	.34100	.51859	70.000	55.000
31.349	19.608	-39.816	-1.2925	.22002	35.000	53.000
5.1011	6.4368	-22.073	1.0126	.76856	65.000	43.000
29.851	16.985	-34.093	-1.7544	.28584-01	33.000	64.000
7.6940	12.539	-24.387	1.6281	1.0504	60.000	8.0000
20.381	-1.1015	-25.148	-2.8215	-.74871	34.000	38.000
12.146	10.212	-18.743	.48390	.16747	55.000	76.000
28.332	24.967	-39.838	.34634	1.0965	48.000	41.000
8.4914	-.65631	-26.837	-.40947	.35149	21.000	60.000
10.389	21.135	-13.613	1.3070	1.1972	87.000	80.000
43.270	24.079	-57.172	-1.5103	.67540	7.0000	52.000
-16.947	-4.8694	-4.9921	2.5732	1.3144	35.000	46.000
25.914	26.189	-19.703	-.90330	.73798	33.000	44.000
6.7679	2.2748	-24.414	.41747	.59619	73.000	45.000
33.525	18.657	-36.615	-2.0219	-.12387	64.000	81.000
23.428	20.713	-43.094	.74217	.90310	15.000	46.000
-4.8769	4.1187	-9.0746	1.3669	1.1309	64.000	25.000
32.147	12.139	-31.395	-2.7633	-.44612	62.000	50.000
20.512	9.7145	-35.812	-.296[illegible]1	.27719-01	63.000	44.000
22.539	12.492	-33.061	-.95648	.31434-01	69.000	54.000
26.659	14.941	-39.260	-.82473	.12849	52.000	55.000
15.641	12.045	-30.015	.13915	.51710	39.000	48.000
13.275	12.229	-22.867	.12694	.62353	58.000	24.000
20.944	3.8308	-28.548	-1.6542	-.39446	59.000	56.000
24.085	12.404	-34.186	-.20555	.27641	43.000	56.000
12.385	12.366	-28.006	.53697	.77521	85.000	47.000
36.283	15.423	-44.703	-2.4073	-.26144	75.000	44.000
21.839	4.8514	-38.241	-1.0778	-.33579	30.000	29.000
4.8377	4.5198	-13.035	.30933	.18275	73.000	69.000
40.170	31.235	-46.838	-1.0052	.50303	49.000	38.000
4.2512	-5.3638	-26.740	-.95005-01	.17277	61.000	50.000
30.398	23.016	-30.475	-1.1237	.16514	35.000	65.000
8.4435	12.340	-28.162	1.4419	1.1120	46.000	58.000
19.532	18.698	-27.158	-.23859	.85805	44.000	73.000
17.247	18.977	-31.385	.76774	1.1899	2.0000	56.000
-6.0950	9.3388	-5.9142	2.0941	1.6305	62.000	65.000
36.216	27.875	-39.153	-1.3871	.74591	85.000	26.000
22.391	-7.2420	-41.162	-2.6111	-.89332	41.000	56.000
15.044	19.336	-20.411	.90889	.53798	75.000	29.000
29.166	6.8975	-41.488	-2.4199	-.49419	33.000	33.000
4.1954	2.1308	-14.983	.54337	.13115	76.000	51.000
39.124	24.984	-43.666	-1.8423	-.18634-01	84.000	31.000
25.276	5.4098	-46.729	-1.2439	-.24428	55.000	41.000
15.746	9.0885	-25.101	-.43994	.21340-01	31.000	51.000
11.765	15.134	-21.581	.80495	.79902	46.000	48.000
18.570	13.604	-28.525	-.83917	.54589	42.000	28.000
11.125	1.7136	-20.602	-.73116	-.12350-01	71.000	56.000
34.510	23.160	-40.442	-1.1996	.11505	50.000	24.000
7.0691	-6.1617	-24.927	-.70234	-.27333	66.000	44.000
32.218	21.235	-33.320	-1.3832	-.14056	69.000	52.000
22.625	9.6426	-41.804	-.60622	.87638-01	80.000	57.000
30.818	17.426	-42.072	-1.2804	.28572-01	.00000	60.000
-10.463	8.3066	-6.2324	3.1175	1.6228	59.000	17.000
29.631	9.9534	-28.447	-3.1001	-.40956	46.000	47.000
14.063	7.8411	-27.314	.32744	.20521	33.000	70.000
14.699	24.868	-24.859	1.2762	1.3457	55.000	53.000
20.463	11.337	-34.342	-.76596	.57313	61.000	41.000
20.506	7.6554	-29.811	-1.1838	-.54265-01	58.000	58.000
23.456	16.021	-34.013	-.19462	.41794	85.000	47.000
31.666	11.166	-45.853	-2.0057	-.27915	69.000	6.0000
15.513	-9.2548	-26.561	-2.2802	-1.2318	72.000	46.000
34.180	22.432	-38.060	-1.0950	-.32335	37.000	51.000
5.8176	7.0712	-26.369	1.1245	.70997	37.000	29.000
13.676	8.5998	-17.411	-.77689	.19814	28.000	32.000
7.5451	6.3952	-15.114	.29452	.38461	58.000	41.000
25.267	15.795	-31.081	-1.1171	.13119	69.000	87.000
30.054	27.341	-46.728	.41263	1.0210	50.000	42.000
3.1472	-.52427	-29.826	.89941-01	.54482	21.000	45.000

SIMCO AND DONA OUTPUT VARIABLES

SIMCO Y1	DONA Y1	SIMCO Y2	DONA Y2
500.00	469.16	3000.0	13925.
576.00	148.21	2500.0	2907.7
768.00	12.584	2500.0	-3381.9
384.00	163.18	2500.0	-3973.4
384.00	40.482	2500.0	-4362.0
576.00	111.07	2500.0	-3229.1
576.00	100.05	2500.0	-3960.8
768.00	122.34	2500.0	-4796.0
576.00	111.97	.00000	-4707.3
576.00	138.53	.00000	-4622.4
576.00	150.08	1050.0	-5337.7
576.00	85.431	4050.0	-5042.2
576.00	180.65	.00000	-4877.2
768.00	53.166	300.00	-5134.4
960.00	176.30	4600.0	-4068.5
576.00	58.827	.00000	-5189.0
576.00	132.34	.00000	-2830.8
576.00	61.542	11600.	-3987.0
576.00	162.58	5850.0	-4831.3
576.00	80.380	6450.0	-4756.4
576.00	80.093	.00000	-4169.4
576.00	240.11	2950.0	-5229.5
576.00	-37.777	.00000	-5538.3
768.00	159.94	1100.0	-2246.3
576.00	48.703	.00000	-3504.5
768.00	193.55	4350.0	-4553.8
576.00	135.11	250.00	-6645.9
576.00	23.968	.00000	-4337.0
576.00	179.22	10200.	-2604.6
576.00	109.15	.00000	-5367.0
576.00	138.88	.00000	-5081.6
768.00	152.00	4450.0	-5481.4
768.00	106.49	.00000	-5485.2
768.00	95.353	7100.0	-4257.7
960.00	124.30	.00000	-3446.3
576.00	134.27	.00000	-5203.6
576.00	90.657	.00000	-4912.2
576.00	207.60	.00000	-5071.5
768.00	127.33	2600.0	-6198.7
768.00	47.954	15150.	-4110.4
768.00	212.56	.00000	-4687.9
960.00	53.940	.00000	-5305.4
576.00	175.36	3950.0	-4682.2
768.00	62.780	.00000	-4944.8
960.00	137.47	4250.0	-4440.2
576.00	114.57	7050.0	-5085.6
576.00	13.422	6900.0	-3674.8
768.00	208.65	1100.0	-3597.0
384.00	134.69	.00000	-5307.6
576.00	93.390	5650.0	-5693.4
768.00	163.64	.00000	-3718.4
768.00	38.237	.00000	-4708.1
960.00	209.32	.00000	-4235.9
768.00	149.66	.00000	-6472.3
768.00	108.25	.00000	-5609.9
768.00	77.699	15500.	-3976.3
960.00	119.33	.00000	-3800.2
576.00	78.456	7400.0	-3647.1
768.00	184.72	.00000	-4825.3
576.00	58.230	.00000	-4513.4
576.00	174.88	.00000	-4577.2
768.00	127.57	10700.	-5482.9
768.00	183.16	.00000	-6307.4
768.00	-20.069	.00000	-4720.4
960.00	176.75	4700.0	-1779.9
576.00	73.605	.00000	-4783.2
768.00	104.97	.00000	-4680.8
576.00	135.25	2050.0	-4279.6
576.00	129.31	.00000	-4914.7
576.00	135.37	20050.	-5216.8
576.00	182.71	.00000	-5537.9
768.00	95.650	.00000	-4799.7
768.00	170.42	.00000	-5123.9
768.00	50.248	.00000	-4973.0
960.00	94.464	1850.0	-3210.5
576.00	49.177	10100.	-2768.3
768.00	142.01	.00000	-3445.8
384.00	170.90	.00000	-6492.6
576.00	62.168	8600.0	-5021.3
576.00	79.234	5850.0	-3225.1

SIMCO AND DONA STATE VARIABLES

SIMCO PSI 1	DONA PSI 1	SIMCO PSI 2	DONA PSI 2	SIMCO PSI 3	DONA PSI 3	SIMCO PSI 4	DONA PSI 4	SIMCO PSI 5	DONA PSI 5
50.000	-120.77	50.000	-55.869	150.00	159.32	50.000	-28.139	50.000	7.1623
.00000	91.819	135.00	-5.7248	191.00	-5.8424	36.333	-2.7176	55.667	-1.9948
.00000	-9.4181	.00000	13.727	191.00	-23.763	36.333	.24907	53.000	.58276
21.000	33.617	1.0000	13.079	184.00	-35.510	38.667	-1.1989	49.667	-.12118
81.000	2.1293	.00000	4.1999	201.00	-17.635	46.667	.30443	40.333	.47157
.00000	18.309	9.0000	17.152	216.00	-21.558	41.667	-.12576	41.000	.56590
6.0000	15.876	86.000	11.692	227.00	-20.795	37.333	-.45715-01	44.333	.58096
92.000	18.758	133.00	17.591	225.00	-28.845	42.000	.12925	51.333	.77818
78.000	16.378	183.00	14.403	179.00	-29.299	45.667	.15746	56.333	.88679
117.00	21.917	26.000	11.358	120.00	-31.959	50.667	-1.0236	55.333	.22787
140.00	25.914	33.000	13.752	73.000	-37.277	57.667	-.96987	52.667	.88521-01
14.000	11.853	21.000	9.8916	110.00	-25.321	57.000	.34180	49.667	.51859
129.00	31.349	84.000	19.608	40.000	-39.816	60.667	-1.2925	52.333	.22002
.00000	5.1011	26.000	6.4368	11.000	-22.073	49.667	1.0126	52.667	.76856
59.000	29.851	.00000	16.965	38.000	-34.093	56.667	-1.7544	50.333	.20584-01
.00000	7.6940	200.00	12.539	5.0000	-24.387	44.333	1.5281	53.333	1.0584
.00000	20.381	.00000	-1.1015	.00000	-25.148	52.667	-2.8215	38.333	-.70871
22.000	12.146	.00000	10.212	143.00	-18.743	42.333	.48390	36.667	.16767
87.000	28.332	.00000	29.967	205.00	-39.838	49.667	.34634	40.667	1.0965
5.0000	8.4914	72.000	-.65631	284.00	-26.837	45.667	-.40947	51.667	.35149
.00000	10.389	145.00	21.135	265.00	-13.613	41.333	1.3070	59.000	1.1972
204.00	43.270	112.00	29.079	237.00	-57.172	52.000	-1.5103	60.333	.67548
.00000	-16.947	57.000	-4.0694	230.00	-4.9921	38.333	2.5732	64.000	1.3144
.00000	25.914	.00000	26.109	217.00	-19.703	43.000	-.90330	59.333	.73798
.00000	6.7679	.00000	2.2748	184.00	-24.418	25.000	.41747	47.333	.59619
89.000	33.525	.00000	18.657	198.00	-36.615	47.000	-2.0219	45.000	-.12357
142.00	23.428	179.00	20.713	139.00	-43.094	56.667	.74217	56.667	.98318
.00000	-4.0769	17.000	4.1187	124.00	-9.0746	50.667	1.3669	57.333	1.1309
.00000	32.157	.00000	12.139	264.00	-31.395	47.667	-2.7633	58.667	-.04612
52.000	28.512	.00000	9.7145	202.00	-35.812	47.000	-.29641	40.333	.27719-01
208.00	22.539	.00000	12.492	139.00	-33.061	63.000	-.95648	39.667	.31434-01
90.000	26.659	109.00	14.941	159.00	-39.260	64.667	-.82473	49.333	.12849
5.0000	15.641	71.000	12.045	107.00	-30.015	61.333	.13915	51.000	.51718
.00000	13.275	53.000	12.229	218.00	-22.867	53.333	.12694	52.333	.62353
.00000	20.944	.00000	3.8308	152.00	-28.548	49.667	-1.8542	42.333	-.39446
79.000	24.685	1.0000	18.404	93.000	-34.186	52.000	-.28555	42.667	.27641
39.000	12.385	80.000	12.366	50.000	-28.806	53.333	.53697	45.333	.77521
138.00	36.283	107.00	15.423	.00000	-44.703	62.333	-2.4073	53.000	-.26164
162.00	21.839	5.0000	4.8514	.00000	-38.241	67.667	-1.0778	49.000	-.33579
85.000	4.8377	.00000	4.5158	215.00	-13.035	63.333	.30933	40.000	.18275
22.000	40.170	108.00	31.235	142.00	-46.838	59.333	-1.0052	47.333	.50303
.00000	4.2512	.00000	-5.3638	93.000	-26.740	50.667	-.95005-01	45.333	.17277
113.00	30.398	114.00	23.016	111.00	-38.475	61.000	-1.1237	32.333	.16514
.00000	8.4435	68.000	12.340	76.000	-28.162	48.333	1.4419	51.000	1.1120
.00000	19.532	111.00	18.698	115.00	-27.158	47.333	-.23859	57.667	.85805
.00000	17.247	157.00	18.977	212.00	-31.385	41.667	.76774	65.333	1.1899
.00000	-6.0950	12.000	9.3368	348.00	-5.9142	30.667	2.0941	62.333	1.6305
.00000	34.216	95.000	27.875	308.00	-39.153	36.000	-1.3871	64.667	.74591
146.00	22.391	.00000	-7.2420	223.00	-41.162	49.667	-2.6111	49.000	-.89332
114.00	15.044	.00000	19.336	295.00	-20.411	62.667	.90889	49.000	.55798
146.00	29.166	.00000	6.6975	220.00	-41.488	67.000	-2.4199	37.000	-.49419
.00000	4.1954	.00000	2.1308	167.00	-14.983	49.667	.54337	39.333	.13115
59.000	34.124	.00000	24.964	111.00	-43.666	61.333	-1.8423	37.667	-.18634-01
119.00	25.278	55.000	5.4096	27.000	-46.729	64.333	-1.2439	45.000	-.24428
30.000	15.746	55.000	9.0885	.00000	-25.101	71.667	-.43994	47.667	.21948-01

.00000	11.765	61.000	15.134	251.00	-21.581	50.667	.80495	47.667	.79902
.00000	18.570	36.000	13.664	205.00	-28.525	44.000	-.83517	46.667	.54589
.00000	11.125	.00000	1.7136	311.00	-20.602	39.667	-.73116	42.333	-.12350-01
94.000	34.510	60.000	23.100	240.00	-40.442	53.000	-1.1996	44.000	.11505
41.000	7.0691	.00000	-6.1617	190.00	-24.927	34.333	-.70234	36.000	-.27333
154.00	32.218	32.600	21.235	124.00	-33.320	62.333	-1.3832	41.333	-.14956
60.000	22.625	40.000	9.6426	[illegible]	-41.804	[illegible]	-.60622	40.000	.87638-01
161.00	30.018	161.00	17.426	189.00	-42.822	.1.667	-1.2004	51.000	.20572-01
.00000	-10.463	111.00	8.3066	189.00	-6.2324	49.667	3.1175	56.333	1.6228
.00000	29.631	.00000	9.9534	220.00	-28.047	46.333	-3.1001	44.667	-.00956
.00000	14.063	.00000	7.8411	178.00	-27.313	35.000	.32744	41.333	.20521
.00000	10.899	39.000	24.868	145.00	-24.857	46.000	1.2762	44.667	1.3457
.00000	20.463	144.00	11.337	131.00	-34.342	44.667	-.76596	56.667	.57313
37.000	20.508	11.000	7.6554	70.000	-29.811	49.667	-1.1838	54.667	-.54265-01
130.00	23.458	39.000	18.021	413.00	-34.013	58.000	-.19462	50.667	.41794
202.00	31.668	18.000	11.166	328.00	-45.853	68.000	-2.0057	48.667	-.27915
77.000	15.513	.00000	-9.2548	259.00	-26.561	70.667	-2.2802	37.000	-1.2318
117.00	34.180	.00000	22.432	187.00	-38.060	75.333	-1.0950	53.000	-.32335
.00000	5.8176	.00000	7.0712	150.00	-24.369	59.333	1.1245	34.333	.70997
.00000	13.678	54.000	8.5998	150.00	-17.411	46.667	-.77689	42.000	.19914
.00000	7.5451	.00000	6.3952	328.00	-15.114	32.667	.29452	37.333	.38461
.00000	25.267	13.000	15.795	274.00	-31.001	38.333	-1.1171	34.000	.13119
.00000	30.054	307.00	27.341	205.00	-46.728	49.000	.41263	53.333	1.0218
40.000	3.1472	27.000	-.52427	295.00	-20.326	54.333	.89941-01	56.667	.54482
.00000	10.985	60.000	16.350	391.00	-11.799	43.333	.55890	58.000	.81083

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